

# Emotion Recognition from Facial Expression using General Type-2 Fuzzy Set

Anisha Halder<sup>1</sup>, Rajshree Mandal<sup>1</sup>, Amit Konar<sup>1</sup>, Aruna Chakraborty<sup>2</sup>, R. Janarthanan<sup>3</sup>

<sup>1</sup>Department of Electronics and Tele-Communication Engineering, Jadavpur University  
Kolkata-32, India

halder.anisha@gmail.com, rajshree.mondal@gmail.com, konaramit@yahoo.co.in

<sup>2</sup>Department of Computer Science and Engineering, St. Thomas' College of Engineering and Technology  
Kolkata, India

aruna\_stcet@rediffmail.com

<sup>3</sup>Department of IT, Jaya Engg. College, Chennai  
srmjana\_73@yahoo.com

**Abstract**—Facial expression of a person representative of similar emotions is not always unique. Naturally, the facial features of a subject taken from different instances of the same emotion have wider variations. In presence of two or more facial features, the variation of the attributes together makes the emotion recognition problem more complicated. This variation is the main source of uncertainty in the emotion recognition problem, which has been addressed here in two steps using type-2 fuzzy sets. First a type-2 fuzzy face-space is constructed with the background knowledge of facial features of different subjects for different emotions. Second, the emotion of an unknown facial expression is determined based on the consensus of the measured facial features with the fuzzy face-space. General Type-2 Fuzzy Sets have been used to model the fuzzy face space. The general type-2 fuzzy set involves both primary and secondary membership distributions, which have been obtained here by formulating and solving an optimization problem. The optimization problem here attempts to minimize the difference between two decoded signals: the first one being the type-1 defuzzification of the average primary membership distributions obtained from the  $n$ -subjects, while the second one refers to the type-2 defuzzified signal for a given primary distribution with secondary memberships as unknown. The uncertainty management policy adopted using general type-2 fuzzy set has resulted in a classification accuracy of 96.67%.

**Index Terms** -Emotion Recognition, Facial feature extraction, Type-2 primary membership, Type-2 secondary membership, Fuzzy Face Space.

## I. INTRODUCTION

Emotion recognition is currently gaining importance for its increasing scope of applications in human-computer interactive systems. Several modalities of emotion recognition, including facial expression, voice, gesture and posture have been studied in the literature. However, irrespective of the modality, emotion recognition comprises two fundamental steps involving feature extraction and classification [7]. Feature extraction refers to determining a set of features/attributes, preferably independent, which together represents a given emotional expression. Classification aims at mapping emotional features into one of several emotion classes.

Among the well-known methods of determining human emotions, Fourier descriptor [19], template matching [19], and neural network techniques [3], [4] deserve special mention. Other important works undertaken so far for recognition of emotions from facial expression by selecting suitable features include [15], [17], [18], [19] and by identifying the right classifier include [3], [4], [8], [12], [17], [20], [22].

Emotional features, on many occasions, are subjective. For example, facial expression of a representative emotion varies greatly from person to person. Further, facial expression of a given subject experiencing similar emotions in different situations looks different. Naturally, it is difficult to have a unique set of features for a given emotion, free from both intra- and inter- personal level variations. In this paper, we attempt to consider emotion recognition as a problem of *uncertainty management* with a view to minimizing the effect of intra- and inter- personal level variations in features on emotion classification.

The paper provides an alternative approaches to emotion recognition from an unknown facial expression, when the emotion class of individual facial expression of a large number of experimental subjects is available. The General Type-2 fuzzy set (GT2FS) based approach employs to construct a fuzzy face space, comprising both primary and secondary membership distributions, obtained from known facial expressions of several subjects containing multiple instances of the same emotion for each subject. The emotion class of an unknown facial expression is determined by obtaining maximum support of each class to the given facial expression. The class with the maximum support is the winner. The maximum support evaluation here employs both primary and secondary distributions. Experiments reveal that the classification accuracy of emotion of an unknown person by the GT2FS based scheme is as high as 96%.

The paper is divided into five sections. In section II, we propose the principle of uncertainty management in fuzzy face-space for emotion recognition. Experimental details are given in section III and the performance analysis is done in section IV. Conclusions are listed in section V.



represented by the  $n$  primary memberships, we use the fuzzy meet operation

$$S_c^{\min} = \text{Min}\{\mu_{\bar{A}}^{\text{mod}}(f'_1), \mu_{\bar{A}}^{\text{mod}}(f'_2), \dots, \mu_{\bar{A}}^{\text{mod}}(f'_m)\} \quad (3)$$

$$S_c^{\max} = \text{Min}\{\mu_{\bar{A}}^{\text{mod}^-}(f'_1), \mu_{\bar{A}}^{\text{mod}^-}(f'_2), \dots, \mu_{\bar{A}}^{\text{mod}^-}(f'_m)\} \quad (4)$$

Thus we can say that the unknown subject is experiencing the emotion class  $c$  at least to the extent  $s_c^{\min}$ , and at most to the extent  $s_c^{\max}$ .

To reduce the non-specificity associated with the interval  $S_{c-i}$ , different approaches can be taken. For example, the most conservative approach would be to use lower bound, while the most liberal view would be to use the upper bound of the interval as the support for the class  $c$ . In absence of any additional information, a balanced approach would be to use center of the interval as the support for the class  $c$  by the  $n$  primary memberships to the unknown subject. This idea is supported by Mendel [9] and Lee [28]. We compute the centre,  $S_c$  of the interval  $S_{c-i}$ ,

$$S_c = (s_c^{\min} + s_c^{\max})/2. \quad (5)$$

Thus  $S_c$  is the degree of support that the unknown facial expression is in emotion class  $c$ .

Now to predict the emotion of a person from his facial expression, we determine  $S_c$  for each emotion class. Presuming that there exist  $k$  emotion classes, let us denote them by  $S_1, S_2, \dots, S_k$  for emotion class 1, 2, ...,  $k$ , respectively. Since a given facial expression may convey different emotions with different degrees, we resolve the conflict by ranking the  $S_i$  for  $i = 1$  to  $k$ , and thus determine the emotion class  $r$ , for which  $S_r \geq S_i$  for all  $i$  following the Rule  $R_c$ .

To make the algorithm robust, we consider association of fuzzy encoded measurements with emotion class by considering the weakest reliability of the joint occurrence of the fuzzy measurements, and identify the winning emotion class having this measure of reliability superseding the same of other emotion classes.

### Methodology

We briefly discuss the main steps involved in fuzzy face space construction based on the measurements of  $m$  facial features for  $n$ -subjects, each having  $l$  instances of facial expression for a particular emotion. We need to classify an facial expression of an unknown person into one of  $k$  emotion classes.

1. We extract  $m$  facial features for  $n$  subjects, each having  $l$  instances of facial expression for a particular emotion. The above features are extracted for  $k$ -emotion classes.
2. We construct a fuzzy face space for each emotion class separately. The fuzzy face space for an emotion class comprises a set of  $n$  primary membership and secondary membership distributions for each feature. Thus we have  $m$  groups of  $n$ -primary as well as secondary membership distributions. Each membership curve is constructed

from  $l$ -facial instances of a subject attempted to exhibit a particular emotion in her facial expression by acting.

3. For a given feature  $f'_i$ , we consult each primary and secondary membership curve under a given emotion class, and take the product of primary and secondary membership at  $f'_i = f'_i$ . The resulting membership value obtained for the membership curves for the subject  $w$  is given by

$$\mu_{\bar{A}}^w(f'_i) = \mu_{\bar{A}}^w(f'_i) \times \mu(f'_i, \mu_{\bar{A}}^w(f'_i)) \quad (6)$$

where the parameters have their usual meaning.

Now, for  $w = 1$  to  $n$ , we evaluate  $\mu_{\bar{A}}^w(f'_i)$ , and thus obtain the minimum and the maximum values of  $\mu_{\bar{A}}^w(f'_i)$ , to obtain a range of uncertainty  $[\mu_{\bar{A}}^{\text{mod}}(f'_i), \mu_{\bar{A}}^{\text{mod}^-}(f'_i)]$ . This is repeated for all features under each emotion class.

4. Now for an emotion class  $j$ , we take fuzzy meet operation over the ranges for each feature to evaluate the range of uncertainty for individual emotion class. The meet operation here is computed by taking cumulative  $t$ -norm of  $\mu_{\bar{A}}^{\text{mod}}(f'_i)$  and  $\mu_{\bar{A}}^{\text{mod}^-}(f'_i)$  separately for  $i = 1$  to  $m$ , and thus obtaining  $S_j^{\min}$  and  $S_j^{\max}$  respectively.
5. The support of the  $j$ -th emotion class to the measurements is evaluated by taking average of  $S_j^{\min}$  and  $S_j^{\max}$ , and defining the result by  $S_j$ .
6. Now by using classifier rule, we determine the maximum support offered by all the  $k$  emotion classes, and declare the unknown facial expression to have emotion  $r$ , if  $S_r > S_i$  for all emotion class  $i = 1$  to  $k$ .

### III. EXPERIMENTS DETAILS

In this section, we present the experimental details of emotion recognition using the principles introduced in section II. We here consider 5 emotion classes, (i.e.,  $k=5$ ) including anger, fear, disgust, happiness and relaxation. The experiment is conducted with two sets of subjects: a) the first set of  $n (=10)$  subjects is considered for designing the fuzzy face-space and, b) the other set of 30 facial expressions taken from 6 unknown subjects are considered to validate the result of the proposed emotion classification scheme. Five facial features, (i.e.,  $m=5$ ) have been used here to design the type-2 fuzzy face-space.

We now briefly overview the main steps of feature extraction followed by fuzzy face-space construction and emotion recognition of an unknown subject using the pre-constructed face-space.

#### A. Feature Extraction

Feature extraction is a fundamental step in emotion recognition. This paper considers extraction of features from emotionally rich facial expressions synthesized by the subjects by acting. Existing research results [14], [22] reveal that the most important facial regions responsible for the manifestation

of emotion are the eyes and the lips. This motivated us to select the following features: Left Eye Opening ( $EO_L$ ), Right Eye Opening ( $EO_R$ ), Distance between the Lower Eyelid to the Eyebrow for the Left Eye ( $LEE_L$ ), Distance between the Lower Eyelid to Eyebrow for the Right Eye ( $LEE_R$ ), and the Maximum Mouth opening (MO) including the lower and the upper lips. Fig. 1 explains the above facial features on a selected facial image.

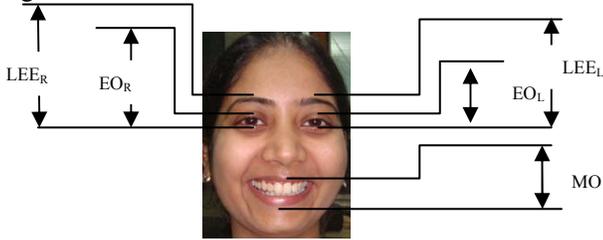


Fig. 1 Facial Features

For extraction of any of the features mentioned above, the first step that needs to be carried out is to separate out the skin and the non-skin regions of the image.

**Estimation of Eye features ( $EO_L$ ,  $LEE_L$ ,  $EO_R$  and  $LEE_R$ ):** To compute the eye features, we first localize the eye region as shown in Fig. 2 (a). A look at Fig. 2 reveals that there is a sharp change in color while moving from the forehead region to the eyebrow region. Thus to detect the location of the eyebrow, we take the average intensity (in three primary color planes) over each row of the image from the top, and identify the row with a maximum dip in all the three planes. This row indicates the top of the eyebrow region (Fig.2 (b)). Similarly, we detect the lower eyelid by identifying the row with a sharp dip in intensity in all the three planes, while scanning the face up from the bottommost row. The location of the top eyelid region is identified by scanning the face up from the marked lower eyelid until a dip in the three color planes are noted together.

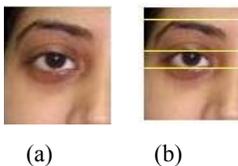


Fig. 2. (a) Localized Eye search region , (b) Detection of eye features

**Estimation of Mouth Opening (MO):** In order to estimate the MO, we first localize the mouth region as shown in Fig. 3.a. Then a conversion from r-g-b to l-a-b color space is undertaken. The k-means clustering algorithm is applied next on this image to get three clusters. The three clusters are: skin, lip and teeth regions. The cluster with the highest intensity variance in l-a-b color space is declared as the lip region. Thus we select the lip cluster (Fig. 3.b) to determine the mouth opening. To obtain the mouth-opening, we take the average intensity of three primary pixel colors, and plot the row-average of such value contributed by each pixel against the row number (Fig. 3.c). It is observed that the width of the zero-crossing zone in Fig. 3.c provides a measure of mouth-opening.

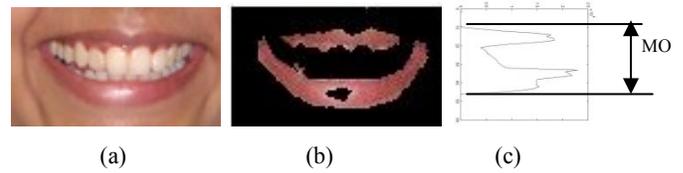


Fig. 3.(a) Mouth search area (b) Lip cluster (c) Graph of average intensity over each row against the row position.

### B. Creating the type-2 fuzzy face-space

The Type-2 fuzzy face-space contains the primary and corresponding secondary membership distributions for each facial feature. Since we have 5 facial features and the experiment includes 5 distinct emotions of 10 subjects, we obtain  $10 \times 5 \times 5 = 250$  primary as well as secondary membership curves. To compute primary memberships, 10 instances of a given emotion is used. These 250 membership curves are grouped into 25 heads, each containing 10 membership curves of ten subjects for a specific feature for a given emotion. Fig. 4 gives an illustration of one such group of 10 membership distributions for the feature  $EO_L$  for the emotion: disgust.

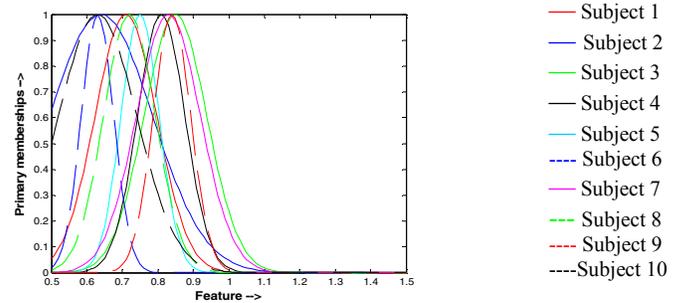


Fig. 4 Membership distributions for emotion disgust and feature  $EO_L$

For each primary membership distribution, we have a corresponding secondary membership distribution. Thus we obtain 250 secondary membership distributions. One illustrative type-2 secondary distributions of subject 1 for the feature  $EO_L$  for the emotion disgust are given in Fig. 5. The axes in the figure represent feature ( $EO_L$ ), primary and secondary membership values as indicated.

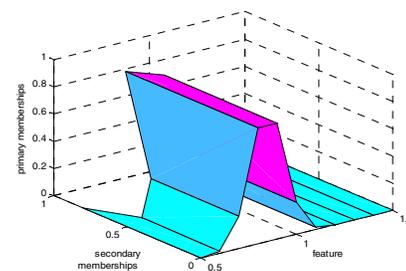


Fig. 5. Secondary Membership curve of Subject 1

### A. Emotion recognition of an unknown facial expression

The emotion recognition problem addressed here attempts to determine the emotion of an unknown person from her facial expression. To keep the measurements in an emotional expression normalized and free from distance variation from the camera focal plane, we construct a bounding box, covering only the face region, and the reciprocal of the diagonal of the bounding box is used as a scale factor for normalization of the measurements. The normalized features obtained from Fig.6 are enlisted in Table I. We now briefly explain the experimental results obtained by our proposed method.



Fig. 6. Facial Image of an unknown subject

TABLE I  
EXTRACTED FEATURES OF FIG. 6

EO <sub>L</sub>	EO <sub>R</sub>	MO	LEE <sub>L</sub>	LEE <sub>R</sub>
0.026	0.026	0.135	0.115	0.115

The GT2FS based recognition scheme considers a fuzzy face space of 5 sets of 10 primary membership distributions as in Fig. 4, and the corresponding secondary membership distributions to the individual primary membership distribution of 5 features obtained from facial expressions carrying 5 distinct emotions for 10 different subjects are determined using curves like Fig. 5.

TABLE II  
CALCULATED TYPE-2 MEMBERSHIP VALUES FOR THE FEATURE :EO<sub>L</sub>  
UNDER EMOTION: DISGUST

Feature	Primary Memberships ( $\mu_{pri}$ )	Secondary memberships ( $\mu_{sec}$ )	$\mu^{mod} = \mu_{pri} \times \mu_{sec}$	Range ( $\min\{\mu^{mod}\}, \max\{\mu^{mod}\}$ )
EO <sub>L</sub>	0.65	0.72	0.468	0.044-0.468
	0.10	0.55	0.055	
	0.15	0.58	0.087	
	0.45	0.68	0.306	
	0.18	0.56	0.1008	
	0.55	0.68	0.374	
	0.08	0.55	0.044	
	0.41	0.63	0.2583	
	0.16	0.53	0.0848	
	0.12	0.59	0.0708	

Table II provides the summary of the primary and secondary memberships obtained for EO<sub>L</sub> for the emotion: disgust. For each feature we obtain 5 Tables like Table II, each one for a given emotion. Thus for 5 features, we would have altogether 25 such tables. In Table II, we also computed the product of primary and secondary memberships, and then obtain the minimum and maximum of the product to determine its range, as indicated in the last column of Table II.

TABLE III  
CALCULATED FEATURE RANGES AND CENTRE VALUE FOR EACH EMOTION

Emotion	Range of Secondary Membership for Features					Range S <sub>c</sub> <sup>J</sup> after fuzzy Meet operation (centre)
	EO <sub>L</sub>	EO <sub>R</sub>	MO	LEE <sub>L</sub>	LEE <sub>R</sub>	
Anger	0-0.14	0 - 0.17	0.39 - 0.964	0.0034 - 0.814	0.0029- 0.781	0-0.14 (0.07)
Disgust	0.044- 0.468	0.041- 0.531	0-0	0-0.39	0.-0.283	0-0 (0)
Fear	0 - 0.298	0-0.275	0.04- 0.742	0.054- 0.473	0.057- 0.511	0-0.275 (0.1375)
Happiness	0 - 0.555	0-0.604	0.573- 0.910	0.133- 0.851	0.3214- 0.7213	0-0.555 (0.2775)
Relaxed	0 - 0.132	0-0.221	0-0	0.013- 0.458	0.046- 0.552	0-0 (0)

The range for each feature corresponding to individual emotions is given in Table III. For example, the entry (0-0.14) corresponding to the row Anger and column EO<sub>L</sub>, gives an idea about the extent of the EO<sub>L</sub> for the unknown subject matches with known subjects from the emotion class Anger. The results of computing fuzzy meet operation over the range of individual features taken from facial expressions of the subjects under same emotional condition are given in Table III. The average of the ranges along with its centre value is also given in Table III. It is observed that the centre has the largest value (=0.2775) for the emotion: happiness

### IV. PERFORMANCE ANALYSIS

We now analyze the performance of our method with 30 facial expressions, each representing a given emotion. The emotion conveyed in each image is analyzed, and the results are tabulated in the form of confusion matrix [14], [24]. The method yields a confusion matrix given in Table IV. It is clear from Table IV that the relaxation and disgust emotions can be classified with 100% high accuracy followed by happiness (96%). Fear and anger are often confusing, and thus classification accuracy is relatively low for both. Average accuracy of the diagonal entries of Table VI is 96.67%.

TABLE IV  
CONFUSION MATRIX FOR TYPE-2 SECONDARY FUZZY FACE-SPACE WITH ENTRIES IN %

In/Out	Anger	Disgust	Happiness	Fear	Relaxed
Anger	93.33	0	3.33	6.66	0
Disgust	0	100	0	0	0
Happiness	0	0	96.66	0	0
Fear	6.66	0	0	93.33	0
Relaxed	0	0	0	0	100

Table V shows the comparison of our proposed emotion recognition scheme with other existing methods. It is clear from the Table that the proposed method has the highest classification accuracy with respect to the published results.

TABLE V  
COMPARISON OVER OTHER TECHNIQUES

Facial Recognition by	Emotion	Success rate
Multi-Layer Perceptron(MLP) and Radial Basis Function Network (RBFN) [24][25]		73% for MLP, and 65% for RBFN
Fuzzy Neural Network and HMM [3]		Higher than 78%
Using Linear Support Vector Machine [23]		95%
(Type-1) Fuzzy Relational Approach [14]		88.2% for adult male and 92.2 for adult female
Bayesian classifier[27][5]		63% to 71%
Fuzzy Kernel Clustering and Support Vector Machines[28]		90%
Principal Component Analysis (PCA) [6]		Average classification accuracy is 100% on train data and 95.47% on cross validation data.
<b>Our proposed method</b>		<b>96.67%</b>

## V. CONCLUSION

The paper employs GT2FS-based automatic emotion recognition of an unknown facial expression, when the background knowledge about a large face database with known emotion class are available. The GT2FS-based recognition scheme requires type-2 secondary membership distributions, a computation of which by an evolutionary approach is also provided. The scheme first construct a fuzzy face space, and then infer the emotion class of the unknown facial expression by determining the maximum support of the individual emotion classes using the pre-constructed fuzzy face space. The class with the highest support is regarded as the emotion of the unknown facial expression. The scheme, however, takes care of both the inter- and intra-subject level uncertainty, and thus offers a higher classification accuracy for the same set of features. Experimental analysis confirms that the classification accuracy of emotion by employing GT2FS is 96.67%.

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