

# Uncertainty Management in Type-2 Fuzzy Face-Space for Emotion Recognition

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**Abstract**—Manifestation of a given emotion on facial expression is not always unique, as the facial attributes in different instances of similar emotional experiences may vary widely. When a number of facial attributes are used to recognize the emotion of a subject, the variation of individual attributes together makes the problem more complicated. This variation is the main source of uncertainty in the emotion recognition problem, which has been addressed here in two steps using type-2 fuzzy sets. First a type-2 fuzzy face-space is constructed with the background knowledge of facial features of different subjects for different emotions. Second, the emotion of the unknown subject is determined based on the consensus of the measured facial features with the fuzzy face-space. The face-space comprises both primary and secondary membership distributions. The primary membership distributions here have been constructed based on the highest frequency of occurrence of the individual attributes. Naturally, the membership values of an attribute at all except the point of highest frequency of occurrence suffer from inaccuracy, which has been taken care of by secondary memberships. An algorithm for the evaluation of the secondary membership distribution from its type-2 primary counterpart has been proposed. The uncertainty management policy adopted using general type-2 fuzzy set has a classification accuracy of 96.67% in comparison to 88.67% obtained by interval type-2 counterpart only.

**Keywords**—Emotion Recognition, Facial feature extraction, Type-2 primary membership, Type-2 secondary membership

## I. INTRODUCTION

Emotion recognition is currently gaining importance for its increasing scope of applications in human-computer interactive systems. Several modalities of emotion recognition, including facial expression, voice, gesture and posture have been studied in the literature. However, irrespective of the modality, emotion recognition comprises two fundamental steps involving feature extraction and classification [1]. Feature extraction refers to determining a set of features/attributes, preferably independent, which together represents a given emotional expression. Classification aims at mapping emotional features into one of several emotion classes.

Both feature extraction and classification are equally important to determine the performance of an emotion recognition system. Sometimes, a good classification algorithm cannot yield high classification accuracy for poorly selected features. Naturally, selection of good features is a pre-requisite for high classification accuracy. Several methods of emotion recognition from facial expression have been developed over the last three decades. The study undertaken so far aimed at improving the performance of emotion classification by either selecting suitable features [2], [3], [4], [5], [6], [7], [8], [9], or by identifying the right classifier [10], [11], [12], [13], [14], [15], [6], [16], [17].

Emotional features, on many occasions are subjective. For example, facial expression of a representative emotion varies greatly from person to person. Further, facial expression of a given subject experiencing similar instances of a given emotion looks different. Naturally, it is difficult to have a unique set of features for a given emotion, free from both intra- and inter-personal level variations. In this paper, we thus attempted to consider emotion recognition as a problem of *uncertainty management* with an aim to minimize the effect of intra- and inter-personal level variations in features on emotion classification.

Type-2 fuzzy set has the potential to handle both intra- and inter-personal level uncertainty [18], which usually is modeled by the primary and secondary membership distributions of each measurement variable obtained from different sources. This characteristic of type-2 fuzzy sets motivated us to employ it for emotion classification of an unknown person. The principle of type-2 fuzzy emotion classification comprises two fundamental steps. First a type-2 fuzzy face-space, containing primary and secondary membership distributions of facial features for different subjects carrying distinct emotions, is created. Next to determine the emotion of an unknown person, selective facial features of the person are extracted and encoded using type-2 fuzzy sets, and finally an approximate matching algorithm is invoked to compare the fuzzy encoded features with those in the fuzzy face-space. The classification of the given facial expression to one of several emotion classes is determined by a simple but robust fuzzy decision making algorithm. It is indeed

important to note here that the decision making process here considers taking support of individual emotion class to a given set of fuzzy measurements, and declaring the class with the *maximum support* as the winner.

Experiments reveal that the classification accuracy of emotion of an unknown person by the general type-2 fuzzy set based scheme is as high as 96%. When secondary memberships are ignored, and classification is performed with only interval type-2 fuzzy sets, the classification accuracy falls by a margin of 8%. The additional 8% classification accuracy obtained by general type-2 fuzzy set, however, has to pay a price for additional complexity of  $(m \times n \times k)$  multiplications, where  $m$ ,  $n$  and  $k$  denote the number of features/subject, no. of subjects, and no. of emotion classes respectively.

The paper is divided into seven sections. In section II, we propose the principle of uncertainty management in fuzzy face-space for emotion recognition. Section III is concerned with the computation of fuzzy type-2 membership distributions. Section IV provides the main steps for emotion classification with a more emphasis on feature extraction. Experimental details are given in section V. Performance Analysis is given in section VI. Conclusions are listed in section VII.

## II. UNCERTAINTY MANAGEMENT IN FUZZY FACE-SPACE FOR EMOTION RECOGNITION

This section provides a general overview of the proposed scheme for emotion recognition using type-2 fuzzy sets. Here, the emotion recognition problem is considered as uncertainty management in fuzzy space after encoding the measured facial attributes by type-2 fuzzy sets.

Let  $F = \{f_1, f_2, \dots, f_m\}$  be the set of  $m$  facial features. Let  $\mu_A(f_i)$  be the primary membership in  $[0,1]$  of the feature  $f_i$  to be a member of set  $A$ , and  $\mu(f_i, \mu_A(f_i))$  be the secondary membership of the measured variable  $f_i$  in  $[0,1]$ . If the measurement of a facial feature,  $f_i$ , is performed  $p$  times on the same subject experiencing the same emotion, and the measurements are quantized into  $q$  intervals of equal size, we can evaluate the frequency of occurrence of the measured variable  $f_i$  in  $q$  quantized intervals. The interval containing the highest frequency of occurrence then can be identified, and its centre,  $m_i$ , approximately represents the mode of the measurement variable  $f_i$ . The second moment,  $\sigma_i$ , around  $m_i$  is determined, and an exponential bell-shaped (Gaussian) membership function centered on  $m_i$  and with spread  $\sigma_i$  is used to represent the membership function of the random variable  $f_i$ . This function represents the membership of  $f_i$  to be CLOSE-TO the central value,  $m_i$ . It may be noted that a bell-shaped (Gaussian-like) membership curve would have a peak at the centre with a membership value one, indicating that membership at this point is the largest for an obvious reason of having the highest frequency of  $f_i$  at the centre.

On repetition of the above experiment for variable  $f_i$  on  $n$  subjects, each experiencing the same emotion, we obtain  $n$  such membership functions, each one for one individual subject. Naturally, the measurement variable  $f_i$  now has both intra- and inter-personal level uncertainty. The intra-personal level uncertainty occurs due to pre-assumption of the bell-shape (Gaussian distribution) of the membership function, and the

inter-personal level uncertainty occurs due to multiplicity of the membership distribution for  $n$  subjects. Thus a new measurement for an unknown person can be encoded using all the  $n$ -membership curves, giving  $n$  possible membership values, thus giving rise to uncertainty in the fuzzy space.

The uncertainty involved in the present problem has been addressed here by two distinctive approaches, using: i) Interval Type-2 Fuzzy Set (IT2FS), and ii) General Type-2 Fuzzy Set (GT2FS). Naturally, the former approach is simple, but more error-prone as it ignores the intra-personal level uncertainty. The second approach is more robust, and is capable to take care of both the uncertainties. But it needs additional complexity as it involves more computation. Though both the approaches have many common steps, we briefly outline the first one for its simplicity, and then explain the latter without repeating the common steps further.

### A. First Approach

Let  $f_i'$  be the measurement of the  $i$ -th feature for an unknown subject experiencing an emotion of class  $c$ . Now, by consulting the  $n$  known primary membership distributions for a given emotion, we obtain  $n$  primary membership values of the variable. Let they be  $\mu_A^1(f_i')$ ,  $\mu_A^2(f_i')$ , ...,  $\mu_A^n(f_i')$ . Here,  $\mu_A^k(f_i')$  represents the degree of membership of a feature  $f_i'$  for the unknown subject in a fuzzy set  $A$ , interpreted using the known membership distribution of the same feature for the  $k$ -th subject.

We now determine the maximum and minimum of these  $n$  membership values, and thus obtain an interval  $[\mu_A^{\min}(f_i'), \mu_A^{\max}(f_i')]$ , representing the entire span of uncertainty of the measurement variable  $f_i'$  in the fuzzy space, induced by  $n$  subjects. If there exist  $m$  different facial features, then for each feature we would have such an interval, and consequently we obtain  $m$  such intervals given by

$$[\mu_A^{\min}(f_1'), \mu_A^{\max}(f_1')], [\mu_A^{\min}(f_2'), \mu_A^{\max}(f_2')], \dots, [\mu_A^{\min}(f_m'), \mu_A^{\max}(f_m')].$$

Since an emotion is characterized by all of these  $m$  features, to find the overall support of the  $m$  features ( $m$  measurements made for the unknown subject) to the emotion class  $c$  represented by the  $n$  primary memberships, we use the fuzzy meet operation to get the interval  $S_{c-i} = [s_c^{\min}, s_c^{\max}]$ , where

$$s_c^{\min} = \text{Min}\{\mu_A^{\min}(f_1'), \mu_A^{\min}(f_2'), \dots, \mu_A^{\min}(f_m')\},$$

$$\text{and } s_c^{\max} = \text{Min}\{\mu_A^{\max}(f_1'), \mu_A^{\max}(f_2'), \dots, \mu_A^{\max}(f_m')\}.$$

Thus we can say that the unknown subject is experiencing the emotion class  $c$  at least to the extent  $s_c^{\min}$ , and at the most to the extent  $s_c^{\max}$ .

To reduce the non-specificity associated with the interval  $S_{c-i}$ , different approaches can be taken. For example, the most conservative approach would be to use lower bound, while the most liberal view would be to use the upper bound of the interval as the support for the class  $c$ . In absence of any additional information, a balanced approach would be to use center of the interval as the support for the class  $c$  by the  $n$  primary memberships to unknown subject. This idea is supported by Mendel [31] and Lee [32]. Thus, we compute the centre,  $S_c$  of the interval  $S_{c-i}$ ,

$$S_c = (s_c^{\min} + s_c^{\max})/2.$$

Thus  $S_c$  is the degree of support that the unknown subject is in emotion class  $c$ .

Now to predict the emotion of a person from his facial expression, we determine  $S_c$  for each emotion class. Presuming that there exist  $k$  emotion classes, let us denote them by  $S_1, S_2, \dots, S_k$  for emotion class 1, 2, ...,  $k$ , respectively. Since a given facial expression may convey different emotions with different degrees, we resolve the conflict by ranking the  $S_j$  for  $j = 1$  to  $k$ , and thus determine the emotion class  $l$ , for which  $S_l \geq S_j$  for all  $j$ .

To make the algorithm robust, we consider association of fuzzy encoded measurements with emotion class by considering the weakest reliability of the joint occurrence of the fuzzy measurements, and identify the winning emotion class having this measure of reliability superseding the same of other emotion classes.

### B. Second Approach

The second approach employs similar methodology like the first one to reduce inter-personal level uncertainty, but considers secondary memberships to reduce intra-personal level uncertainty as well. The main step in the second approach is to evaluate type-2 secondary membership values from the primary membership distributions. This, of course, is a difficult problem, and is discussed in detail in section III. The intra-personal level uncertainty in primary membership is reduced by correctly adjusting them with their secondary membership by the following transformation.

Let  $f_i^j$  be the measurement of the  $i$ -th feature for an unknown subject. Now, by consulting the  $n$  primary membership functions for a given emotion, we obtain  $n$  primary membership values of the variable given by  $\mu^1_A(f_i^j), \mu^2_A(f_i^j), \dots, \mu^n_A(f_i^j)$ . Let the secondary membership values for each primary membership respectively be  $\mu(f_i^j, \mu^1_A(f_i^j)), \mu(f_i^j, \mu^2_A(f_i^j)), \dots, \mu(f_i^j, \mu^n_A(f_i^j))$ . Since the secondary memberships denote the degree of accuracy of the primary memberships, the uncertainty in a primary membership distribution can be reduced by multiplying each primary membership value by its secondary membership. Thus the modified primary membership values are given by

$$\begin{aligned} \text{mod. } \mu^1_A(f_i^j) &= \mu^1_A(f_i^j) \times \mu(f_i^j, \mu^1_A(f_i^j)), \\ \text{mod. } \mu^2_A(f_i^j) &= \mu^2_A(f_i^j) \times \mu(f_i^j, \mu^2_A(f_i^j)), \dots, \\ \text{mod. } \mu^n_A(f_i^j) &= \mu^n_A(f_i^j) \times \mu(f_i^j, \mu^n_A(f_i^j)), \end{aligned}$$

where  $\text{mod. } \mu^j_A(f_i^j)$  denotes the modified primary membership value for  $j$ -th subject. The rest of the procedure in approach 2 is similar to approach 1 with replacement of  $\mu^j_A(f_i^j)$  by  $\text{mod. } \mu^j_A(f_i^j)$  for all  $j = 1$  to  $n$ .

### III. FUZZY TYPE-2 MEMBERSHIP EVALUATION

This section provides an introduction to type-2 membership evaluation [20], [21], [22]. Classical fuzzy sets consider primary membership distribution  $\mu_A(x)$  of a fuzzy linguistic variable  $x$ , in a fuzzy set  $A$ , where  $x \in X$  and  $A \subseteq X$ . Usually  $\mu_A(x)$  is defined by an expert in a specialized domain.

However, it has been found that different experts assign different membership distribution for the variable  $x$ . Consider for example, a fuzzy linguistic variable age in universe AGE, where age belongs to 0 to 120 years. Let YOUNG be a fuzzy set in the universe AGE. Now, the membership distribution  $\mu_{\text{YOUNG}}(\text{age})$  can be assigned by different people in different forms. Considering a Gaussian type distribution, for  $\mu_{\text{YOUNG}}(\text{age})$ , we note that the peak of the distribution may be fixed at 18, 20 or 22 years depending on the relative preference of the individual. Such membership distributions suffer from two distinct type of uncertainty. The first type of uncertainty, called the intra-personal level uncertainty, is due to approximate values of membership within a given distribution assigned by an expert. The inter-personal level uncertainty refers to relative variations among the membership distributions for a given value of age.

Although theoretically very sound, type-2 fuzzy set has limitedly been used over the last two decades because of the users' inadequate knowledge to correctly assign the secondary memberships. This paper, however, overcomes this problem by extracting type-2 membership distribution from its type-1 counterpart by an evolutionary algorithm. A brief outline to the construction of secondary membership distribution is given in this section.

Intuitively, when an expert assigns a grade of membership distribution, she is relatively more certain to determine the location of the peaks and the minima over the distribution, but may not have enough background to correctly assign the membership values in the rest. Presuming that the membership values at the peak and the minima are close to 1, we attempt to compute secondary memberships at the rest of the secondary distribution. The following assumptions are used to construct an objective function, which needs to be minimized to obtain the solution of the problem.

1. Let  $x=x_p$  and  $x=x_q$  be two successive optima (peak/minimum) on the primary membership distribution  $\mu_A(x)$ . Then at any point  $x$  lying between  $x_p$  and  $x_q$ , the secondary membership  $\mu(x, \mu_A(x))$  will be smaller than both  $\mu(x_p, \mu_A(x_p))$  and  $\mu(x_q, \mu_A(x_q))$ .
2. The fall-off in secondary membership at a point  $x$  away from a peak/minimum  $\mu(x_p, \mu_A(x_p))$  is exponential, given by

$$\mu(x, \mu_A(x)) = \mu(x_p, \mu_A(x_p)) \cdot \exp(-|x - x_p|) \quad (1)$$

The secondary membership at any point  $x$  between two consecutive optima at  $x=x_p$  and  $x=x_q$  in the primary membership is selected from the range  $[\alpha, \beta]$ , where

$$\alpha = \mu(x_p, \mu_A(x_p)) \cdot \exp(-|x - x_p|) \quad (2)$$

$$\text{and } \beta = \mu(x_q, \mu_A(x_q)) \cdot \exp(-|x - x_q|) \quad (3)$$

3. The defuzzified signal obtained on averaging the primary membership distributions at all  $x$  from  $n$  sources, is equal to the defuzzified signal obtained from type-2 primary and secondary distributions. The logic behind this, though apparent, is briefly explained here. The inter-

personal level uncertainty involved in the averaged membership distribution is the minimum with respect to individual primary distribution, as the uncertainty is also averaged at a given value of  $x$ , and is thus reduced.

4. The secondary membership at two consecutive values of  $x$  separated by a small positive  $\delta$  has a small difference. This is required to avoid sharp changes in the secondary grade.

Given the primary membership distributions from  $n$  sources be  $\mu_{A1}(x), \mu_{A2}(x), \dots, \mu_{An}(x)$ , then the average membership distribution which represents a special form of fuzzy aggregation is given by

$$\mu_A(x) = \frac{\sum_{i=1}^n \mu_{Ai}(x)}{n}, \forall x \quad (4)$$

i.e., at each position of  $x=x_j$ , the above membership aggregation is employed to evaluate a new composite membership profile  $\mu_A(x)$ . The defuzzified signal obtained by the centroidal method [23] from the averaged primary membership distribution is given by

$$\bar{x} = \frac{\sum_{\forall x} x \mu_A(x)}{\sum_{\forall x} \mu_A(x)} \quad (5)$$

Further, the type-2 centroidal defuzzified signal obtained from the  $i$ -th primary and secondary membership distributions is given by

$$\bar{x} = \frac{\sum_{\forall x} x \mu_{Ai}(x) \cdot \mu_{Ai}(x, \mu_{Ai}(x))}{\sum_{\forall x} \mu_{Ai}(x) \cdot \mu_{Ai}(x, \mu_{Ai}(x))} \quad (6)$$

Using assumptions 3 and 4, we construct a performance index  $J$  given by (7)

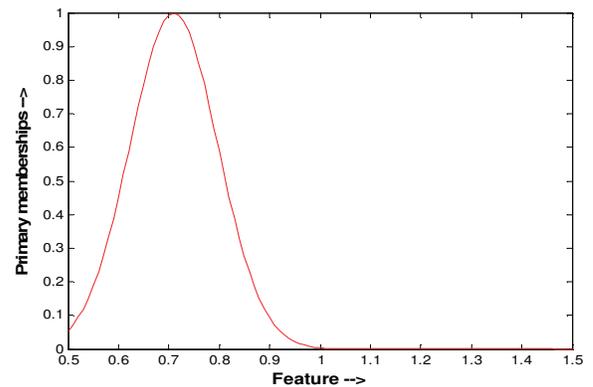
$$J = (\bar{x} - x)^2 + \sum_{x=x_1}^{x_{i-1}} \{\mu((x + \delta), \mu_A(x + \delta)) - \mu(x, \mu_A(x))\}^2 \quad (7)$$

The secondary membership evaluation problem, now, transforms to minimization of  $J$  by selecting  $\mu(x, \mu_A(x))$  from a given range  $[\alpha, \beta]$ , where  $\alpha$  and  $\beta$  are the secondary memberships at the two optima in secondary distribution around the point  $x$ . Expressions (2) and (3) are used to compute  $\alpha$  and  $\beta$  for each  $x$  separately.

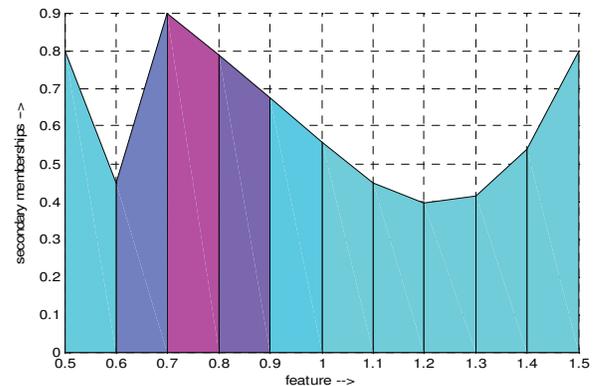
Any derivative-free optimization algorithm can be used to minimize  $J$ , and obtain  $\mu(x, \mu_A(x))$  at each  $x$  except the optima on the secondary distribution. Differential Evolution (DE) is one such derivative-free optimization algorithm, which has fewer control parameters, and has outperformed the well-known binary coded Genetic Algorithm and Particle Swarm Optimization algorithms with respect to standard benchmark functions. Further, DE is simple and has only a few lines code, which motivated us to employ it for the above optimization problem. The details of DE are not given here for lack of space. Readers may have it from the standard literature [24]. The main steps of the secondary membership evaluation procedure are outlined below.

1. Obtain the averaged primary membership distribution  $\mu_A(x)$  from the primary distributions  $\mu_{Ai}(x)$  obtained from  $n$  sources, i.e.,  $i=1$  to  $n$ . Evaluate  $\bar{x}$ , and also  $\bar{x}$  for a selected primary membership distribution  $\mu_{Ai}(x)$  using (5) and (6) respectively.
2. Find the optima on  $\mu_{Aj}(x)$  for a given  $j$ . Let the set of  $x$  corresponding to the optima be  $S$ . Set the secondary membership  $\mu(x, \mu_{Aj}(x))$  to 0.99 (close to one) for all  $x \in X$ .
3. For each  $x \in X$ , where  $x \notin S$ , identify the optima closest around  $x$  from  $S$ . Let them be located at  $x=x_p$  and  $x=x_q$ , where  $x_p < x < x_q$ . Determine  $\alpha$  and  $\beta$  for each  $x$ , given by (2) and (3).
4. For each  $x$ , where  $\mu(x, \mu_A(x))$  lies in  $[\alpha, \beta]$ , minimize  $J$  by DE.
5. Obtain  $\mu(x, \mu_A(x))$  for all  $x$  after the DE converges.
6. Repeat step 2 onwards for all  $j$ .

An illustrative plot of secondary membership distribution for a given primary is given in Fig. 1



(a)



(b)

Fig. 1.(a) Primary distribution (b) Secondary distribution from the primary .

#### IV. METHODOLOGY

In this section, we present the methodology of emotion recognition using the principles introduced in section II and III. We here consider 5 emotion classes, (i.e.,  $k=5$ ) including anger, fear, disgust, happiness and relaxation. We consider  $n=10$  subjects to characterize the type-2 membership distributions. To study the performance of the proposed scheme, we consider 30 facial expressions taken from 6 unknown persons. Five facial features, (i.e.,  $m=5$ ) have been used here to design the type-2 fuzzy face-space. The whole process of emotion recognition of an unknown person from his/her facial expression is divided into six main steps, as outlined below.

1. Facial feature extraction,
2. Type-2 primary membership evaluation from  $n$ -curves, one for each known subject,
3. Type-2 secondary membership evaluation corresponding to  $n$  primary membership values,
4. Reducing intra-personal level uncertainty by re-defining primary membership as the product of primary and secondary membership,
5. Evaluation of interval  $S_c^j$  and finally  $S_c^j$  for  $k$  competitive emotion classes using modified primary memberships,
6. Determining the winning emotion class  $l$  by selecting  $S_c^l$ , where  $S_c^l$  is the largest among  $S_c^j$  for emotion class  $j$ .

The explanation of all the steps except step 1 has already been outlined in section II. So, we now give an overview to step 1 only.

In order to extract the facial features from emotionally expressive facial images, it is essential to list down the most important features of the face. From earlier research results [19], [17], it is evident that the most important regions for recognizing the emotion of a person are the eyes and lips. This motivated us to select the following features: Left Eye Opening ( $EO_L$ ), Right Eye Opening ( $EO_R$ ), Distance between the Lower Eyelid to Eyebrow for Left Eye ( $LEE_L$ ), Distance between the Lower Eyelid to Eyebrow for Right Eye ( $LEE_R$ ), and Maximum Mouth opening (MO) including lower and upper lips. Fig. 2 explains the above facial features on a selected facial image.

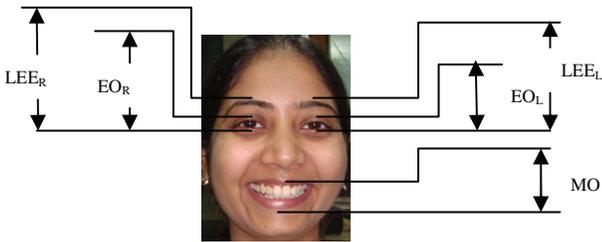


Fig. 2 Facial Features

For extraction of any of the features mentioned above, the first step that needs to be carried out is to separate out the skin and the non-skin regions of the image.

**Estimation of Eye features ( $EO_L$ ,  $LEE_L$ ,  $EO_R$  and  $LEE_R$ ):** To estimate the eye features, we first localize the eye region as

shown in Fig. 3 (a). A look at Fig. 3 reveals that there is a sharp change in color while moving from the forehead region to the eyebrow region. Thus to detect the location of the eyebrow, we take the average intensity (in three primary color planes) over each row of the image from the top, and identify the row with a maximum dip in all the three planes. This row indicates the top of the eyebrow region (Fig.3b). Similarly, we detect the lower eyelid by identifying the row with a sharp dip in intensity in all the three planes, while scanning the face up from the bottommost row. The location of the top eyelid region is identified by scanning the face up from the marked lower eyelid until a dip in the three color planes are noted together.

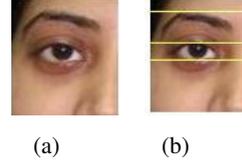


Fig. 3. (a) Localized Eye search region , (b) Detection of eye features

**Estimation of Mouth Opening (MO):** In order to estimate the MO, we first localize the mouth region as shown in Fig. 4(a). Then a conversion from rgb to lab color space is undertaken. The k-means clustering algorithm is applied next on this image to get three clusters. The three clusters are: skin cluster, lip cluster and teeth cluster. We select the lip cluster (Fig. 4(b)) to determine the mouth opening. To obtain mouth-opening, we take the average intensity of three primary pixel colors, and plot the row-average of such value contributed by each pixel against the row number (Fig. 4c). It is observed that the width of the zero-crossing zone in Fig. 4c provides a measure of mouth-opening.



Fig. 4.a

Fig. 4.b

Fig. 4.c

Fig. 4. (a) Mouth search area (b) Lip cluster (c) Graph of average intensity over each row against the row position.

#### V. EXPERIMENTAL DETAILS

The experiment is conducted in Jadavpur University with two sets of subjects: a) the first set of 10 subjects ( $n=10$ ) is considered for designing the fuzzy face-space and, b) the other set of 30 facial expressions taken from 6 unknown subjects is considered to validate the result of the proposed emotion classification scheme. The experiment thus consists of two distinct phases as indicated in the next two sub-sections.

##### A. Creating the type-2 fuzzy face-space

Type-2 fuzzy face-space contains both primary and secondary membership distributions for each facial feature. Since we have 5 facial features and the experiment includes 5 distinct emotions of 10 subjects, we obtain  $10 \times 5 \times 5 = 250$  primary membership curves. These 250 membership curves are grouped into 25 heads, each containing 10 membership curves of ten subjects for a specific feature representing a given emotion. Fig. 5 gives an illustration of one such group of 10

membership distributions for the feature  $EO_L$  for the emotion: disgust.

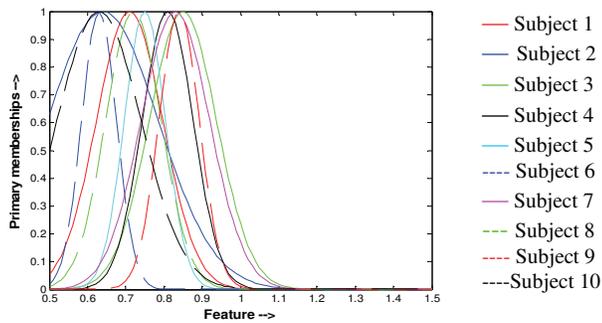


Fig. 5 Membership distributions for emotion disgust and feature  $EO_L$

For each primary membership distribution, we have a corresponding secondary membership distribution. Thus we obtain 250 secondary membership distributions. Two illustrative type-2 secondary distributions for subject 1 and 2 for the feature  $EO_L$  for emotion disgust are given in Fig. 6. The axes in the figure represent feature ( $EO_L$ ), primary and secondary membership values as indicated.

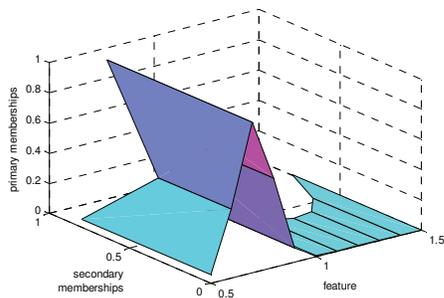


Fig. 6a Secondary Membership curve of Subject1

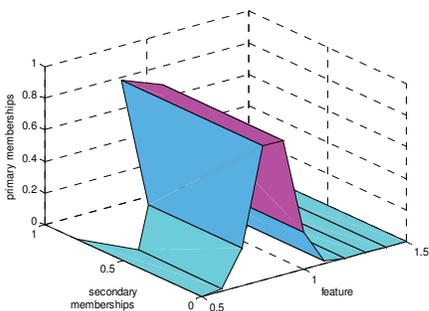


Fig. 6.b Secondary Membership curve of Subject2

### B. Emotion recognition of an unknown subject

Suppose we want to recognize the correct emotion class of the facial image in fig. 7.



Fig. 7 Facial Image of an unknown subject

First, the facial features are extracted as mentioned in section IV. The extracted features are self-normalized by dividing individual feature obtained in a given emotional state by its value in the relaxed state. This nullifies the effect of the distance variation while image capturing. The extracted features for Fig.7 are enlisted in Table I.

TABLE I. EXTRACTED FEATURES OF FIG. 7

$EO_L$	$EO_R$	MO	$LEE_L$	$LEE_R$
0.636	0.6371	1.77	0.969	0.968

Now, for each facial feature, we consult 5 sets of 10 primary membership distributions as in Fig. 5, each corresponding to one distinct emotion of 10 subjects. Thus we obtain 10 primary membership values of that feature to fall in a given emotion class. Secondary memberships corresponding to a given feature and obtained primary membership are determined using curves like Fig. 6. Table II provides the summary of the primary and secondary memberships obtained for  $EO_L$  for the emotion disgust.

TABLE II. CALCULATED MEMBERSHIP VALUES FOR FEATURE : $EO_L$  EMOTION: DISGUST

Feature	Primary Memberships ( $\mu_{pri}$ )	Secondary memberships ( $\mu_{sec}$ )	$\mu_{pri} \times \mu_{sec}$	Range after intersection
$EO_L$	0.1562	0.6371	0.0980	0.0056-0.8011
	0.6531	0.6039	0.3940	
	0.0983	0.5566	0.0545	
	0.9471	0.8460	0.8011	
	0.0983	0.5513	0.0540	
	0.2496	0.5961	0.1487	
	0.0100	0.5566	0.0056	
	0.9000	0.8513	0.7662	
	0.6241	0.8539	0.5290	
0.5134	0.5724	0.2947		

In Table II, we computed the product of primary and secondary memberships, and then obtain the minimum and maximum of the product to determine its range, as indicated in the last column of Table II.

For each feature we obtain 5 Tables like Table II, each one for a given emotion. Thus for 5 features, we would have altogether 25 such tables. The range for each feature corresponding to individual emotions is given in Table III. Now, we obtain a new range with lower (upper) boundary equal to the intersection of the lower (upper) range boundaries for different features under a given emotion. This measure, enumerated in the last column of Table III, indicates an interval of fuzzy certainty about the joint occurrence of the features in a given facial expression representing a specific emotion class.

The center of the interval has minimum uncertainty, and so we compared them, and finally determine the winning emotion with the largest centre. It is clear from Table III that the centre value is maximum for emotion happiness. So, we regard the emotion portrayed in Fig. 7 as Happiness.

TABLE III. CALCULATED FEATURE RANGES AND CENTRE VALUE FOR EACH EMOTION

Emotion	Range of Features					Range $S_e^j$ after Intersection (centre)
	$EO_L$	$EO_R$	$MO$	$LEE_L$	$LEE_R$	
Anger	0-0.199	0 - 0.239	0.439 -0.964	0.0034 - 0.8145	0.0027-0.7813	0-0.239 (0.1195)
Disgust	0.0056-0.8011	0.0021-0.798	0-0	0-0.39	0.0011-0.261	0-0 (0)
Fear	0 - .298	0-0.1751	0.004-0.912	0.024-0.673	0.077-0.715	0-0.1751 (0.0875)
Happiness	0 - 0.555	0.0011-0.604	0.673-0.975	0.1343-0.8151	0.3214-0.7213	0-0.555 (0.2775)
Relaxed	0 - 0.132	0-0.221	0-0	0.013-0.458	0.046-0.552	0-0 (0)

## VI. PERFORMANCE ANALYSIS

We now analyze the performance of our method with 30 facial expressions, each representing a given emotion. The emotion conveyed in each image is analyzed, and the results are tabulated in the form of confusion matrix [14], [19], [25]. It may be noted that in section II, we introduced two methods for emotion recognition. The first method yields a confusion matrix given in Table IV, while the confusion matrix obtained by method 2 is given in Table V. It is clear from Table V that the relaxation and disgust emotion can be classified with 100 % high accuracy followed by happiness (96%). Fear and anger are often confusing, and thus classification accuracy is relatively low for both.

TABLE IV. CONFUSION MATRIX FOR TYPE-2 PRIMARY FUZZY FACE-SPACE WITH ENTRIES IN %

In/Out	Anger	Disgust	Happiness	Fear	Relaxed
Anger	83.33	0	10	16.66	0
Disgust	0	93.33	0	0	0
Happiness	3.33	3.33	86.66	3.33	0
Fear	13.33	0	3.33	80	0
Relaxed	0	3.33	0	0	100

TABLE V. CONFUSION MATRIX FOR TYPE-2 SECONDARY FUZZY FACE-SPACE WITH ENTRIES IN %

In/Out	Anger	Disgust	Happiness	Fear	Relaxed
Anger	93.33	0	3.33	6.66	0
Disgust	0	100	0	0	0
Happiness	0	0	96.66	0	0
Fear	6.66	0	0	93.33	0
Relaxed	0	0	0	0	100

A look at Table IV and V reveals that the percentage accuracy obtained in Table-V is better than the same in Table-IV. It may be recalled that Table-V employed both primary and secondary memberships (i.e., GT2FS), while Table-IV used

only primary membership distributions for n-sources (i.e., IT2FS). This justifies the significance of GT2FS in emotion recognition. Average accuracy obtained by ranking the diagonal entries of Table IV and V are 88.66% and 96.67% respectively.

Table VI shows the comparison of our proposed emotion recognition scheme with other existing methods. It is clear from the Table that the proposed method has the highest classification accuracy with respect to the published results.

TABLE VI. COMPARISON OVER OTHER TECHNIQUES

Facial Emotion Recognition by	Success rate
Multi-Layer Perceptron(MLP) and Radial Basis Function Network (RBFN) [26][27]	73% for MLP, and 65% for RBFN
Fuzzy Neural Network and HMM [10]	Higher than 78%
Using Linear Support Vector Machine [17]	95%
(Type-1) Fuzzy Relational Approach [19]	88.2% for adult male and 92.2 for adult female
Bayesian classifier[28][29]	63% to 71%
Fuzzy Kernel Clustering and Support Vector Machines[30]	90%
Principal Component Analysis (PCA) [25]	Average classification accuracy is 100% on train data and 95.47% on cross validation data.
<b>Our proposed method</b>	<b>96.67%</b>

## VII. CONCLUSION

The paper proposed a simple and time-efficient scheme for emotion recognition from a pre-constructed type-2 fuzzy face-space. Experiments reveal that the classification accuracy of emotion by considering GT2FS is as high as 96.67%. The accuracy falls off by 8% when the solution is obtained by using IT2FS formulation. The classical rule based method for emotion classification [19] depends largely on the relational matrix used to represent implication relations. In the present context, the emotion analysis is performed intentionally on the fuzzy encoded measurement space to make the system performance robust.

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