A Structured Approach to Fuzzy Abduction Based on Contraposition Property of Propositional Logic

Aruna Chakraborty, Amit Konar and Atulya K. Nagar

Abstract- **The paper proposes a new approach to fuzzy abduction by extending the contraposition rule of propositional logic. The main difficulty of fuzzy abduction is due to the restriction of single antecedent clause in a fuzzy production rule. When more than one antecedent clause occurs jointly in a fuzzy rule, the membership value of the derived antecedent clauses become equal, which should not be the case in general. The paper overcomes this problem and demonstrates a simple but elegant method to compute premises in a multi-chained reasoning system.**

I. INTRODUCTION

The principle of abduction attempts to infer a fact p from a given rule: if p then q and an observed phenomenon q. Unfortunately, in classical first ordered propositional logic, we cannot derive p from the fact q and the rule: if p then q. However, abduction is permissible in the logic of fuzzy sets for its inherent capability of partial matching. For example, given a fuzzy production rule: if x is A then y is B, and an observed phenomenon y is $B^{/}$, then by the principle of abduction we can derive x is $A^{/}$, where the fuzzy sets $A^{/}$ and $B^{/}$ are close enough to A and B respectively. The abduction works fine in fuzzy logic as long as there exists a single antecedent clause like *x is A* in the production rule. However, when there exists more than one conjunctive antecedent clause, the principle of abduction returns equal membership distribution for all the antecedent clauses (premises) [1]. This is a fundamental problem in fuzzy abduction. Although many engineering problems such as diagnosis, historical time series prediction etc. require abduction, it is restrictively used for the above fundamental problem.

This paper attempts to overcome this problem by transforming fuzzy production rules with multiple antecedents and consequents by extending the well known contraposition theorem [6] of propositional logic. Such transformation results in new fuzzy production rules with negated conjunctive consequences of the original rule as antecedents, and disjunction of the negated antecedents of the original rule as the new consequents. Naturally, given the membership functions for the observed consequences, one can easily evaluate the membership functions of the antecedent clauses by fuzzy relational algebra.

The paper further demonstrates that the fuzzy implication relation $R_1(x, y)$ for the rule: if x is A then y is B is same as $R_2(y, x)$ for the rule: if y is not B then x is not A with respect to Lukasiewicz implication function. This property holds even when there exists a number of antecedent and consequent clauses in the given rule. Thus, for evaluation of the membership of x is not $A^{/}$ from $R_2(y, x)$ and the membership distribution of y is not $B^{/}$, one need not spend extra computational time to evaluate $R_2(y, x)$ as $R_1(x, y) = R_2(y, x)$. The computation of the membership of x is $A^{/}$ then can easily be evaluated from the computed membership distribution of x is not $A^{/}$ by fuzzy complementation. The same policy is adopted here in abduction, when a rule has multiple antecedent and consequent clauses.

Given a relation R(x, y) for the rule: if x is A then y is B, and the membership distribution for y is $B^{/}$ (where $B^{/} \approx$ B), to evaluate the membership distribution for x is $A^{/}$(where $A^{/} \approx$A), we need to compute $R^{-1}(x, y)$. In classical fuzzy relational algebra [14], $R^{-1}(x, y)$ is computed by taking transposition of R(x, y). In the literature on fuzzy sets [2], [3], the authors have shown more accurate inverse calculation of a relation R(x, y). Naturally when R^{-1} is required for abduction, we should prefer to use more accurate results for R^{-1} rather than presuming $R^{-1} = R^{T}$ [24]. In this paper, we need not compute R^{-1}.

Several attempts have been endeavored to handle the abduction problem [13], [15]-[20], [22], [23]. In early 1980's, researchers were keen to formulate the abductive reasoning problem [4] in terms of conditional probability of events. In the 1990's researchers employ graphical models such as Petri nets for backward (abductive) reasoning [5]. However, because of the fundamental problem introduced above, none of the existing techniques can handle the problem of abduction in its full spirit.

II. TRANSFORMATION OF FUZZY RULES TO ELIMINATE THE OCCURRENCE OF MULTIPLE ANTECEDENTS IN PRODUCTION RULES

In classical propositional logic, the propositional rules include AND connectives in the left hand and OR connectives in the right hand side of the implication (if-

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then) operator [6]. The general format of the rules is given by

$$p_1,p_2,p_3,\ldots\ldots,p_n \rightarrow q_1,q_2,\ldots,q_m \qquad (1)$$

where $p_1, p_2,\ldots, p_n$ and $q_1, q_2, \ldots, q_m$ are atomic propositions [6] in the antecedent and consequent clause respectively of the given propositional rule, and comma in the left(right) hand side denotes AND (OR) connectives.

It can be shown using propositional logic that if any atomic term (i.e., a proposition with or without a negation sign) is transposed to the other side of the if-then (→) operator (with changes in sign), then the resulting rule would be logically equivalent with the original rule. As an example, if all terms of the left and the right hand sides are transposed, then the resulting rule becomes

$$\neg q_1, \neg q_2, \ldots, \neg q_m \rightarrow \neg p_1, \neg p_2, \neg p_3, \ldots\ldots, \neg p_n. \qquad (2)$$

Theorem 1 below proves that (1) and (2) are logically equivalent.

Theorem 1**:** Given the primal form of the propositional statement

$$p_1 \wedge p_2 \wedge p_3 \wedge \ldots\ldots\ldots \wedge p_n \rightarrow q_1 \vee q_2 \vee \ldots\ldots. \vee q_m$$

the dual of which given by

$$\neg q_1 \wedge \neg q_2 \wedge \ldots\ldots \wedge \neg q_m \rightarrow p_1 \vee p_2 \vee p_3 \vee \ldots\ldots. \vee p_n$$

is logically equivalent, i.e., logical interpretations [7] for both the primal and the dual forms are equal.

Proof. Given

$$p_1 \wedge p_2 \wedge p_3 \wedge \ldots\ldots\ldots \wedge p_n \rightarrow q_1 \vee q_2 \vee \ldots\ldots. \vee q_m$$

$$\equiv \neg(p_1 \wedge p_2 \wedge p_3 \wedge \ldots\ldots\ldots \wedge p_n) \vee (q_1 \vee q_2 \vee \ldots\ldots. \vee q_m)$$

$$\text{(Since } p \rightarrow q \equiv \neg p \vee q)$$

$$\equiv (\neg p_1 \vee \neg p_2 \vee \neg p_3 \vee \ldots\ldots\ldots \vee \neg p_n) \vee (q_1 \vee q_2 \vee \ldots\ldots. \vee q_m)$$

$$\text{(By De Morgan's Theorem)}$$

$$\equiv (q_1 \vee q_2 \vee \ldots\ldots. \vee q_m) \vee (\neg p_1 \vee \neg p_2 \vee \neg p_3 \vee \ldots\ldots\ldots \vee \neg p_n)$$

$$\equiv \neg\{\neg(q_1 \vee q_2 \vee \ldots\ldots. \vee q_m)\} \vee (\neg p_1 \vee \neg p_2 \vee \neg p_3 \vee \ldots \vee \neg p_n)$$

$$\text{(Double Negation)}$$

$$\equiv \neg\{\neg q_1 \wedge \neg q_2 \wedge \ldots \wedge \neg q_m\} \vee (\neg p_1 \vee \neg p_2 \vee \neg p_3 \vee \ldots \vee \neg p_n)$$

$$\text{(By De Morgan's Theorem)}$$

$$\equiv \neg q_1 \wedge \neg q_2 \wedge \ldots\ldots. \wedge \neg q_m \rightarrow \neg p_1 \vee \neg p_2 \vee \neg p_3 \vee \ldots\ldots\ldots \vee \neg p_n$$

$$\text{(Since } \neg p \vee q \equiv p \rightarrow q)$$

Hence, the primal and the dual are logically equivalent. □

In fuzzy logic, we have fuzzy production rules of the following general format:

If x_1 is A_1 and x_2 is A_2 and ………and x_n is A_n

Then y_1 is B_1 or y_2 is B_2 or………or y_m is B_m. (3)

Extending the above rule similar to (2), we obtain:

If y_1 is not B_1 and y_2 is not B_2 and …and y_m is not B_m

Then x_1 is not A_1 or x_2 is not A_2 or … or x_n is not A_n. (4)

If we have k number of fuzzy production rules with multiple antecedent clauses, then we can modify all such rules using the transformation (4). Such transformation eliminates the AND connectives from the rule, and thus the problem of fuzzy abduction that results in same fuzzy distribution to the antecedent clauses in a rule [1] thus can be overcome. The principle of fuzzy abduction can be performed easily on the transformed rules. Example below illustrates the construction of a fuzzy OR graph representing the transformed rules.

III. PRINCIPLE OF ABDUCTION FOR A SINGLE RULE SYSTEM

In this section, we present the principle of abduction with a single rule having multiple antecedent and a single consequent clause. However, before introducing the principle, we consider an interesting property of Lukasiewicz implication function [25], given in Theorem 2, which will be used throughout the paper.

Theorem 2: Given a primal rule: if x is A then y is B and its dual if y is not B then x is not A. The Lukasiewicz implication relation $R_1(x.y)$ defined on X×Y for the binary rule is same as $R_2(y,x)$ defined in Y×X for the dual rule for $x \in X$ and $y \in Y$.

Proof. The Lukasiewicz implication function for the primal rule is given by

$$R_1(x,y) = Min[1,(1-\mu_A(x)+\mu_B(y))]$$

The Lukasiewicz implication function for the dual rule is given by

$$R_2(y,x) = Min[1,(1-(1-\mu_B(y))+ (1-\mu_A(x)))]$$
$$= Min[1,(1-\mu_A(x)+\mu_B(y))]$$

$\therefore R_1(x,y) = R_2(y,x)$.

Hence the theorem follows. □

COROLLARY 1: If R_1 and R_2 are in matrix form then the primal and dual implication relations support

$$R_1 = R_2^{T}.$$

Proof: By Theorem 2, we obtain

$$R_1(x, y) = R_2(y, x).$$

Thus it follows that $R_1 = R_2^{T}$. □

It may be noted that the Klnee-Diens and Wu implication relations also support the statement of Theorem 2 and Corollary 1.

A. Abduction with single rule

Suppose for the given rule: If x is A and y is B then z is C, the membership distribution $\mu_A(x)$ and $\mu_B(y)$ and $\mu_C(z)$ are given. Let us now onwards write $\mu_A(x)$ by A(x), $\mu_B(y)$ by B(y), and $\mu_C(z)$ by C(z). We construct fuzzy relations in X×Z and Y×Z, where x∈X, y∈Y and z∈Z. The relational matrices are

$$R_1(x, z) = A^{T} \circ C,$$

$$R_2(y, z) = B^{T} \circ C,$$

where T above a matrix denotes its transpose, and " ∘ " denotes max-min composition operator [8].

Now, suppose we are given the transformed rule if z is not C then x is not A or y is not B, and we need to evaluate $\mu_{\neg A}(x)$, $\mu_{\neg B}(y)$, and $\mu_{\neg C}(z)$, hereafter denoted by $\overline{A}(x), \overline{B}(y)$ and $\overline{C}(z)$ respectively. Then

$$\overline{A}(x) = \overline{C}(z) \circ R_2(z, x).$$

In matrix form

$$\overline{A} = \overline{C}(z) \circ R_1^{T} \text{ [by Corollary 1].}$$

Now to compute A(x) from $\overline{A}(x)$, we use

$$A(x) = 1 - \overline{A}(x).$$

$$= 1 - \overline{C}(z) \circ R_1^{T}.$$

Similarly, we compute

$$B(x) = 1 - \overline{B}(x) = 1 - \overline{C}(z) \circ R_2^{T}.$$

B. Numerical Example

Consider the rule: if height is TALL and weight is MODERATE then speed is HIGH.

Let

$$\mu_{TALL}(height) = A(height) = \begin{matrix} 5' & 6' & 7' \\ [0.5 & 0.6 & 0.7] \end{matrix}$$

$$\mu_{MODERATE}(weight) = B(weight) = \begin{matrix} 50kg & 60kg & 70kg \\ [\ 0.5 & 0.9 & 0.6\ ] \end{matrix}$$

$$\mu_{HIGH}(speed) = C(speed) = \begin{matrix} 5m/s & 6m/s & 8m/s \\ [\ 0.2 & 0.5 & 0.9\ ] \end{matrix}$$

$$\therefore R_1(height, speed)$$

$$= A^{T} \circ C$$

$$= \begin{bmatrix} 0.5 \\ 0.6 \\ 0.7 \end{bmatrix} \circ \begin{bmatrix} 0.2 & 0.5 & 0.9 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.5 & 0.5 \\ 0.2 & 0.5 & 0.6 \\ 0.2 & 0.5 & 0.7 \end{bmatrix}$$

$$R_2(weight, speed)$$

$$= B^{T} \circ C$$

$$= \begin{bmatrix} 0.5 \\ 0.9 \\ 0.6 \end{bmatrix} \circ \begin{bmatrix} 0.2 & 0.5 & 0.9 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.5 & 0.5 \\ 0.2 & 0.5 & 0.9 \\ 0.2 & 0.5 & 0.6 \end{bmatrix}$$

Suppose we are given $\overline{C}(speed) = \begin{matrix} 5m/s & 6m/s & 8m/s \\ [\ 0.2 & 0.9 & 0.8\ ] \end{matrix}$, we want to determine $A(height)$.

We compute $\overline{A}(height) = \overline{C}(speed) \circ R_1^{T}$

$$= \begin{bmatrix} 0.8 & 0.1 & 0.2 \end{bmatrix} \circ \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.5 & 0.5 & 0.5 \\ 0.5 & 0.6 & 0.7 \end{bmatrix}$$

$$= [0.2 \quad 0.2 \quad 0.2]$$

$$So, A(height) = 1 - \overline{A}$$

$$= \begin{matrix} 5m/s' & 6m/s & 8m/s \\ [0.8 & 0.8 & 0.8]. \end{matrix}$$

Similarly,

$$\bar{B}(weight) = \bar{C}(speed) \circ R_2^{\,T}$$

$$= \begin{bmatrix} 0.8 & 0.1 & 0.2 \end{bmatrix} \circ \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.5 & 0.5 & 0.5 \\ 0.5 & 0.9 & 0.6 \end{bmatrix}$$

$$= [0.2 \quad 0.2 \quad 0.2]$$

and $B(weight) = 1 - \bar{B}$. □

$$= [0.8 \quad 0.8 \quad 0.8].$$

IV. ABDUCTION WITH MULTIPLE CHAINED RULES

Consider three fuzzy production rules as follows.

Rule 1: If x is A and y is B Then z is C.
Rule 2: If z is $C^{/}$ and w is D Then u is E.
Rule 3: If u is $E^{/}$ and v is V then p is F.

A fuzzy AND graph (Fig. 1) can be constructed by combining the above rules, where the nodes in the graph represent fuzzy propositions, and the directed arcs denote antecedent-consequent, dependence relationships.

The transformation of the above rules by (4) yields:

Rule 4: If z is not C Then. x is not A or y is not B
Rule 5: If u is not E Then z is not $C^{/}$ or w is not D
Rule 6: If p is not F Then u is not $E^{/}$ or v is not V.

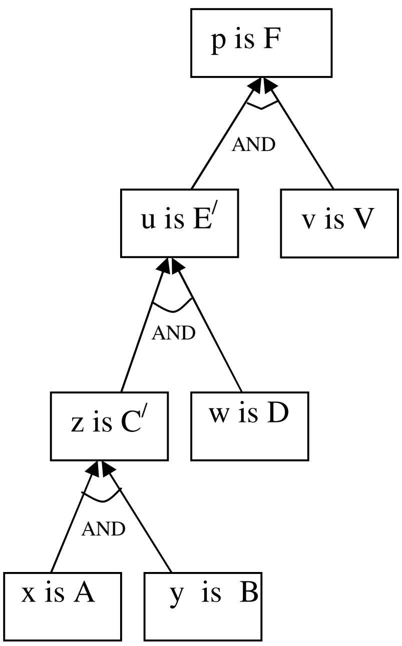

Fig. 1. The fuzzy AND graph representing the firing sequence of three rules in order from the leaves upstream up to the root.

A directed acyclic graph [23], representing the order of firing sequence of the rules starting from the root node downstream up to the leaves, is given in Fig. 2. Here, the connectivity from a parent to its children satisfies implication relation, and the OR operator against an arc denote that the parent-child-1 implication, and is independent of parent-child-2 implication. Such independence would exist for each parent to children implications, as in propositional logic, for any three propositions p, q, r,
$p \rightarrow q \vee r \vee s$ is equivalent to $(p \rightarrow q) \vee (p \rightarrow r) \vee (p \rightarrow s)$.

Suppose, we have the following implication relations: $R_1(x, z)$, $R_2(y, z)$, $R_3(z, u)$, $R_4(w, u)$, $R_5(u, p)$ and $R_6(v, p)$ defined on $X \times Z$, $Y \times Z$, $Z \times U$, $W \times U$, $U \times P$ and $V \times P$, where x, y, z, w, u, p, and v are fuzzy linguistic variables in universes X, Y, Z, W, U, F and V respectively.

Given the following Lukasiewicz implication relations:

$$R_1(x, z) = Min[1, (1 - A(x) + C^{/}(z))]$$

$$R_2(y, z) = Min[1, (1 - B(y) + C^{/}(z))]$$

$$R_3(z, u) = Min[1, (1 - C^{/}(z) + E^{/}(u))]$$

$$R_4(w, u) = Min[1, (1 - D(w) + E(u))]$$

$$R_5(u, p) = Min[1, (1 - E^{/}(u) + F(p))]$$

$$and\ R_6(v, p) = Min[1, (1 - V(v) + F(p))]$$

where $A(x) = \mu_A(x), B(y) = \mu_B(y), C(z) = \mu_C(z),$

$D(w) = \mu_D(w), E(u) = \mu_E(u), V(v) = \mu_V(v)$

and $F(p) = \mu_F(p)$.

Further, let

$\overline{E^{/}} = \mu_{\neg E^{/}}(u),$

$\bar{V} = \mu_{\neg V}(u),$

$\overline{C^{/}} = \mu_{\neg C^{/}}(z),$

$\bar{D} = \mu_{\neg D}(w),$

$\bar{A} = \mu_{\neg A}(x),$

$\bar{B} = \mu_{\neg B}(y).$

We now evaluate (see Fig. 2):

$$R_5^{/}(p,u) = Min[1,(1-\mu_{\neg F}(p)+\mu_{\neg E^{/}}(u))]$$

$$= Min[1,(1-(1-\mu_F(p))+1-\mu_{E^{/}}(p))]$$

$$= Min[1,(1-E^{/}(u)+F(p))] = R_5(u,p)$$

Similarly,

$$R_6^{/}(p,v) = Min[1,(1-V(v)+F(p)))] = R_6(v,p)$$

$$R_3^{/}(u,z) = Min[1,(1-C^{/}(z)+E^{/}(u)))] = R_3(z,u)$$

$$R_4^{/}(u,w) = Min[1,(1-D(w)+E(u)))] = R_4(w,u)$$

$$R_1^{/}(z,x) = Min[1,(1-A(x)+C^{/}(z)))] = R_1(x,z)$$

$$R_2^{/}(z,y) = Min[1,(1-B(y)+C^{/}(z)))] = R_2(y,z)$$

Given the membership distribution of p is not F, we can evaluate the membership of x is not A and y is not B, w is not D and v is not V by the following steps.

Step 1: Compute:

$$\overline{E^{/}} = \overline{F} \circ R_5^{/} = \overline{F} \circ R_5^T$$

$$\overline{V} = \overline{F} \circ R_6^{/} = \overline{F} \circ R_6^T$$

Step 2: Compute:

$$\overline{C^{/}} = \overline{E^{/}} \circ R_3^{/} = \overline{E^{/}} \circ R_3^T$$

$$\overline{D} = \overline{E^{/}} \circ R_4^{/} = \overline{E^{/}} \circ R_4^T$$

Step 3: Compute:

$$\overline{A} = \overline{C^{/}} \circ R_1^{/} = \overline{C^{/}} \circ R_1^T$$

$$\overline{B} = \overline{C^{/}} \circ R_2^{/} = \overline{C^{/}} \circ R_2^T$$

In matrix form

$$R_1^{/} = R_1^T, R_2^{/} = R_2^T, R_3^{/} = R_3^T, R_4^{/} = R_4^T, R_5^{/} = R_5^T$$

$$and\ R_6^{/} = R_6^T.$$

We now re-complement the membership distribution to interpret about the premises of the primal rules. This is done by the following fuzzy complementation operation.

$$A = 1 - \overline{A}$$

$$B = 1 - \overline{B}$$

$$D = 1 - \overline{D}$$

$$and \quad V = 1 - \overline{V}.$$

The concept introduced above with fuzzy rules can be generalized with n-rules (n>3), but not given here for space limitation.

V. ABDUCTION WITH RULES HAVING MULTIPLE ANTECEDENTS AND CONSEQUENTS

In this section, we consider abduction with rules having Multiple antecedent clauses. As an example, let us consider the following two chained rules:

1. If x_1 is A_1 and x_2 is A_2
   Then y_1 is B_1 or y_2 is B_2.

2. If y_1 is $B_1^{/}$ and z_1 is C_1
   Then w_1 is D_1 or w_2 is D_2.

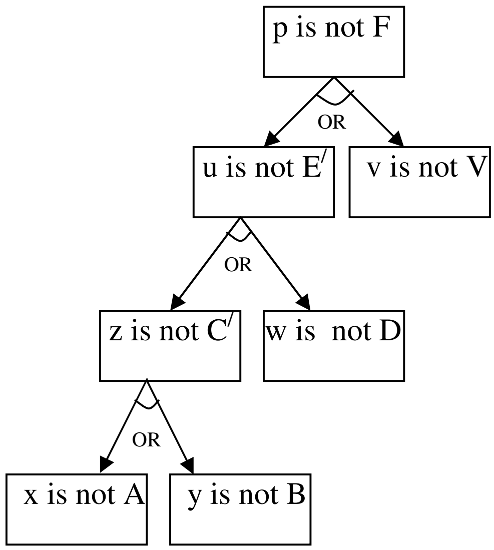

Fig.2: The order of firing sequence of the rules starting from the root node.

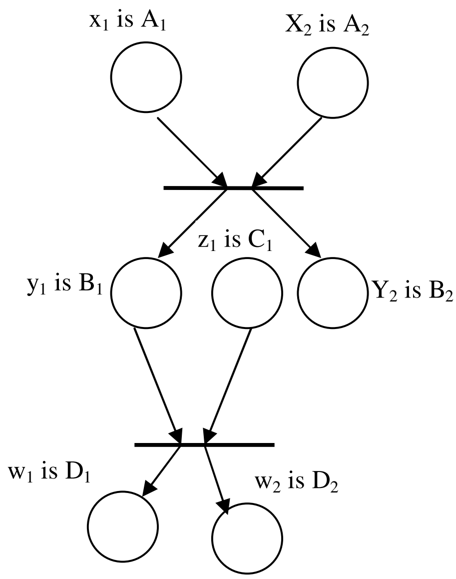

Fig. 3. A primal Petri like net.

The above set of primal rules can be represented by a Petri net like diagram (Fig. 3) [9],[10], where the circles denote places, representing fuzzy propositions and the transitions (bars) denote rules. The nomenclature of Petri net is used here only for representational advantages. No properties of classical Petri nets [11] need to satisfy here.

The transformed rule can be expressed in Petri net as in Fig. 4

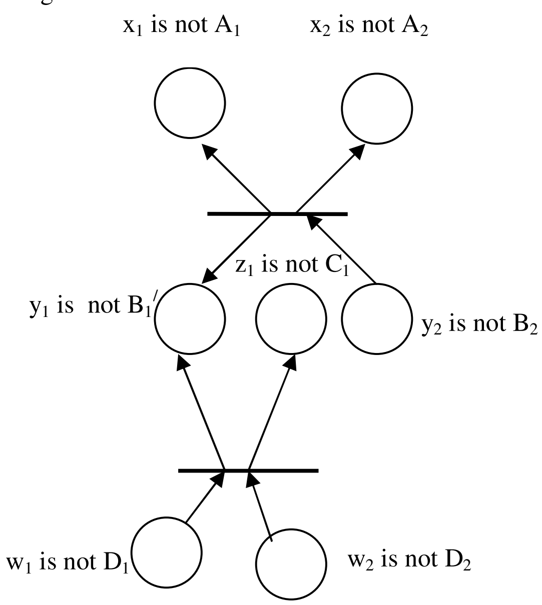

Fig. 4: The dual Petri net corresponding to the primal net of Fig. 3.

The rules 1 and 2 can be expressed like

1. If y_1 is not B_1 and y_2 is not B_2
   Then x_1 is not A_1 or x_2 is not A_2.

2. If w_1 is not D_1 and w_2 is not D_2
   Then y_1 is not $B_1^{/}$ or z_2 is not C_1.

Suppose, the membership distributions for w_1 is not D_1 and w_2 is not D_2 and y_2 is not B_2 are provided. We want to evaluate the membership distribution of x_1 is A_1, and x_2 is A_2 and z_1 is C_1.

We first construct relational matrices $R_1(w_1, w_2; y_1)$ $R_2(w_1, w_2; z_1)$, $R_3(y_1, y_2; x_1)$ and $R_4(y_1, y_2; x_2)$ [8].

Let us symbolize

$$\overline{D}1 = \mu_{\neg D_1}(w_1), \overline{D}2 = \mu_{\neg D_2}(w_2)$$

$$\overline{B_1^{/}} = \mu_{\neg B_1^{/}}(y_1), \overline{C}1 = \mu_{\neg C_1}(z_1),$$

$$\overline{B_2} = \mu_{\neg B_2}(y_2)$$

$$\overline{A_1} = \mu_{\neg A_2}(x_1), \overline{A_2} = \mu_{\neg A_2}(x_2),$$

$$Then\ \overline{B_1^{/}} = (\overline{D}1 \wedge \overline{D}2) \circ R_1(w_1, w_2; y_1)$$

where $(\overline{D}1 \wedge \overline{D}2)$ returns the component-wise minimum of the two row vectors $(\overline{D}1$ and $\overline{D}2)$.

$$\overline{C_1} = (\overline{D}1 \wedge \overline{D}2) \circ R_2(w_1, w_2; z_1)$$

$$\overline{A_1} = (\overline{B_1^{/}} \wedge \overline{B}2) \circ R_3(y_1, y_2; x_1)$$

$$\overline{A_2} = (\overline{B_1^{/}} \wedge \overline{B}2) \circ R_4(y_1, y_2; x_2)$$

Consequently,

$$A_1 = 1 - \overline{A}1$$

$$A_2 = 1 - \overline{A}2 and$$

$$C_1 = 1 - \overline{C}1.$$

Thus we can evaluate the membership distributions of x_1 is A_1, x_2 is A_2 and y_1 is C_1 from the known distributions of w_1 is not D_1, w_2 is not D_2, and y_2 is not B_2. For space limitation, we cannot give numerical illustration to the above reasoning problem.

VI. Performance Evaluation

In this section, we studied the relative performance of the proposed scheme with classical method of fuzzy abduction. For convenience of our analysis, we consider a fuzzy rule: if height is TALL then speed is HIGH, where the membership distribution for height is TALL and speed is HIGH are provided as follows. Let

$$A = \mu_{TALL}(height) = \overset{5^{/}\ \ 6^{/}\ \ 7^{/}}{[0.3\ \ 0.6\ \ 0.9]}$$

$$B = \mu_{HIGH}(speed) = \overset{7m/s\ \ 8m/s\ \ 10m/s}{[\ 0.2\ \ 0.4\ \ 0.7\ ]}$$

With the above membership distributions, we now construct a relational matrix R for the given rule using Lukasiewicz implication function.

$$
\begin{array}{cc} & \begin{array}{ccc} 7 & 8 & 10\ m/s \end{array} \\ \begin{array}{c} 5' \\ R = 6' \\ 7' \end{array} & \begin{bmatrix} 0.9 & 1.0 & 1.0 \\ 0.6 & 0.8 & 1.0 \\ 0.3 & 0.5 & 0.8 \end{bmatrix} \end{array}
$$

Let $B' = B = [0.2\quad 0.4\quad 0.7]$. We now attempt to obtain A' ($\approx$A) using classical method.

$A' = B' \circ R^T$

$= B \circ R^T$

$$
= [0.2 \quad 0.4 \quad 0.7] \circ \begin{bmatrix} 0.9 & 1.0 & 1.0 \\ 0.6 & 0.8 & 1.0 \\ 0.3 & 0.5 & 0.8 \end{bmatrix}
$$

$$
= [0.7 \quad 0.7 \quad 0.7].
$$

Now, we evaluate A' by our proposed scheme. Here,

$$
\overline{A'} = \overline{B'} \circ R^T
$$

$$
= [0.8 \quad 0.6 \quad 0.3] \circ \begin{bmatrix} 0.9 & 1.0 & 1.0 \\ 0.6 & 0.8 & 1.0 \\ 0.3 & 0.5 & 0.8 \end{bmatrix}
$$

$$
= [0.8 \quad 0.6 \quad 0.5].
$$

Consequently,

$$
A' = [0.2 \quad 0.4 \quad 0.5].
$$

Since a correct abduction should return $A' = A$, therefore, we need to evaluate the error of the resulting inferences with respect to $A' = A$ using Euclidean norm. Let E_1 and E_2 be the Euclidean norms of A' with respect to A in classical and proposed reasoning methodology respectively. Thus,

$$
\begin{aligned}
E_1 &= \| (0.7 \quad 0.7 \quad 0.7) - (0.3 \quad 0.6 \quad 0.9)\| \\
&= \| (0.4 \quad 0.1 \quad -0.2)\| \\
&= \sqrt{0.21}.
\end{aligned}
$$

$$
\begin{aligned}
E_2 &= \|(0.2 \quad 0.4 \quad 0.5) - (0.3 \quad 0.6 \quad 0.9)\| \\
&= \|(-0.1 \quad -0.2 \quad -0.4)\| \\
&= \sqrt{0.21}.
\end{aligned}
$$

It is thus apparent that for rules with single antecedent clause, the propose scheme gives same error norm with respect to classical mode of reasoning. However, the significance of this work becomes prominent, when the rules used for abduction have more than one antecedent clauses. In other words, the equality in Euclidean norm ensures that the proposed mode of reasoning is as accurate as the classical one, but the limitations of the classical single antecedent clause based reasoning can be overcome by the proposed scheme.

VII. Conclusions

The paper proposed a novel approach to fuzzy abduction by extending one fundamental property of classical propositional logic. Such extension eliminates the scope of AND-connectives from the antecedent part of fuzzy production rules, and thus simplifies the abductive reasoning problem. It may be noted that classical fuzzy abduction with production rules employing more than one antecedent clause often suffers from a computational problem with a result of equal membership distribution for all the antecedent clauses. The transformation employed in the current paper overcomes this limitation, and consequently computes the fuzzy membership distribution of all the antecedent clauses, which together gives an interpretation to the abductive reasoning problem.

One interesting advantage of the present work is to eliminate the need for fuzzy inverse relations. Inverse computation of relational matrix involves significant computational time [2], [3], and thus in many real time reasoning systems such as intelligent diagnosis, abduction employing fuzzy inverse is no longer suited. This paper proposes a new approach to abductive reasoning that does not require inverse computation.

It may be added here that the fuzzy relations can be constructed using any typical implication functions such as Mamdani's, Godel's [25] and many others. However, in this paper we employed Lukasewicz type implication function, which is insensitive to the proposed transformation of propositional logic.

The abductive reasoning algorithm presented here can be applied to engineering/medical diagnostic problems [12], [21], where the root node of the dual tree/graph represents the defect in a physical system, whereas the leaf/terminal nodes indicate the possible defects in the components/modules of the system. Given the membership profile of the defect in the physical system (root node), the proposed computational methodology provides us an opportunity to evaluate the membership distribution in the component/module level of the system without the need for computing fuzzy inverse relations.

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