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Prediction of the Interactive Dynamics of Stimulated Emotions: Chaos, Limit Cycles and Stability

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Abstract

The paper attempts to model the interactive dynamics of competitive/co-operative emotions, aroused by audio-visual stimulus. Parameter variations of the dynamics results in three specific dynamic behavior including Chaos, Limit cycles and stability. An analysis of the dynamics yields the parametric conditions for Stability and Chaos. Known audio-visual stimulus is used to predict the emotive state of the dynamics from the external manifestation of emotions in the facial expressions of the subjects. Such prediction helps in early detection of Trauma and Epilepsy.

1. Introduction

Since the beginning of the new millennium, there has been a considerable progress in detection and modeling of human emotions. The existing works on emotion detection employ classical neural network [1], [2]-[4], fuzzy logic [5], and statistical methods [6],[3], to uniquely identify a specific emotion of a subject from his/her facial expression [1].Traces of new methods for emotion detection from voice [7], EEG [8], and fMRI [8], images are also seen in the current literature in machine intelligence.

Emotions are the response of the human mind to external stimulus. Naturally, arousal of specific emotions depends on the involved phenomena in the external stimulus. Usually aroused emotions take some time for dispersal. Consequently, significant changes in the stimulus may cause arousal of different emotion with a trace of the previously aroused emotion in the psychological mind. The paper studies competitive/cooperative behavior in the dynamics of the multiple emotions. Identification of a dynamics for competitive / co-operative emotions is an interesting open problem, and determination of parameters of such dynamics is of further concern.

One way to arouse emotions is to excite the psychological brain with suitable audio visual stimulus, responsible for generation of specific emotions. In our early studies [9] we already undertook several methods to experimentally determine the appropriate audio-visual samples for excitation of specific emotions. In this paper, we submit audio visual movie clips to subjects to arouse mixed emotions (i.e. co-existence of more than one emotion), and attempt to determine the parameters of the emotional dynamics from the facial extracts of the subjects. We, however, assume that the subjects participating in the experiments are conducive to the experimental environment.

An analysis of emotional dynamics reveals three possible behaviors; Chaos, Limit cycles and Stability in the emotional states. The conditions for Limit cycles and Chaos have been derived. Consequently, when the parameters of the dynamics are extracted from the facial expression of a subject stimulated with movie clips, we can determine the dynamic behavior of his/her emotional states from the conditions of Chaos and Stability.

The paper is derived into six main sections. In Section -2 we propose a new model for the mixed emotional dynamics. Section-3 is devoted to the analysis of Stability and Chaos. The experimental framework and determination of facial extracts is undertaken in Section -4. Prediction of the state of psychological mind of the subject from his facial expression is presented in Section-5. Conclusions are listed in section-VI.

2. Proposed Model for Chaotic Emotional Dynamics

There exist two types of emotions: simple and complex. A simple emotion includes only one

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emotional state, while a complex emotion includes more than one concurrent emotional state. The latter has been described in this paper by a competitive /co-operative emotional dynamics. Main emphasis of the paper is given on the growth of an emotional state in the influence of co-operation and competition of other emotional states [7]. There exists work on feedback models of emotional dynamics, cited in [6], but the approach undertaken here is new and unknown in the domain of emotional intelligence. Stimulated brain imaging, including Functional Magnetic Resonance Imaging (fMRI) [8], and Positron Emission Tomography undertaken in [10, 11] supports the response of the mathematical model of emotional dynamics presented in this paper[9].

Let X_i, X_j and X_k be three representative emotional states that describe the concentration of individual emotions. Suppose X_j for some j co-operates with X_i, while X_k, for some k, competes with X_i. In other words, the growth rate of X_i will be accelerated with an increase in X_j, but will be decelerated with an increase in X_k. Assuming that there exist m number of co-operative emotional states like X_j and n competitive emotional state like X_k, we can represent the dynamic behavior of emotional state X_i by the following differential equation.

$$\frac{dX_i}{dt} = a_{ii}X_i(1-\frac{X_i}{K}) + \sum_{\exists j} b_{ji}X_i(1-\exp(-\beta_{ji}X_j)) - \sum_{\exists k} c_{ki}X_i(1-\exp(-\lambda_{ik}X_k)) \quad (1)$$

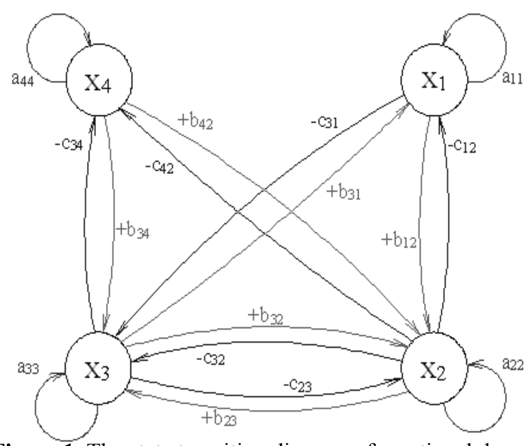

Figure 1. The state transition diagram of emotional dynamics with four emotional states X_1 through X_4.

The 1st term in the R.H.S. of the above equation corresponds to self-growth of emotional state X_i. Here, a_{ii} denotes the inertial co-efficient that regulates the self-growth of X_i. The factor (1- X_i/K) is a controlling term that selects the sign of intrinsic growth rate a_{ii}. For instance when X_i< k, the first term in the right hand side is positive, when X_i =k the 1st term becomes zero, and when X_i> K, it becomes negative. In other words, X_i is allowed to increase up to a level of K, and a fall-off in the growth rate in X_i starts once it exceeds K. The 2nd term represents the co-operation between emotion X_j and X_i for some j. It is indeed important to note that the 2nd term takes into account the co-operation of X_i with a growing X_j. The third term on the other hand represents competition of X_i with growing X_k for some K. The parameters: β_{ji} and λ_{ik} control the growth of X_j and X_k respectively.

The dynamics of the system shown in Figure 1 is presented here by equations (2) – (5).

$$\frac{dX_1}{dt} = a_{11}X_1(1-\frac{X_1}{K}) + b_{31}X_1(1-\exp(-\beta_{31}X_3)) - c_{12}X_1(1-\exp(-\lambda_{21}X_2)) \quad (2)$$

$$\frac{dX_2}{dt} = a_{22}X_2(1-\frac{X_2}{K}) + b_{12}X_2(1-\exp(-\beta_{12}X_1)) + b_{42}X_2(1-\exp(-\beta_{42}X_4)) - b_{32}X_2(1-\exp(-\beta_{32}X_3)) - c_{23}X_2(1-\exp(-\lambda_{32}X_3)) \quad (3)$$

$$\frac{dX_3}{dt} = a_{33}X_3(1-\frac{X_3}{K}) + b_{23}X_3(1-\exp(-\beta_{23}X_2)) - c_{32}X_3(1-\exp(-\lambda_{23}X_2)) - c_{31}X_3(1-\exp(-\lambda_{13}X_1)) - c_{34}X_3(1-\exp(-\lambda_{43}X_4)) \quad (4)$$

$$\frac{dX_4}{dt} = a_{44}X_4(1-\frac{X_4}{K}) + b_{34}X_4(1-\exp(-\beta_{34}X_3)) - c_{42}X_4(1-\exp(-\lambda_{24}X_2)) \quad (5)$$

where the parameters have their usual meaning as discussed above.

3. The Lyapunov Exponent Analysis for the Detection of Chaos

A discrete approximation of the model (2)-(5) can be reconstructed by replacing

$$\frac{dX_i}{dt} = \frac{X_i(t+1) - X_i(t)}{(t+1)-t} = X_i(t+1) - X_i(t)$$

for i = 1 to 4,

The equation (2) can be rewritten as,

$$X_1(t+1) = X_1(1+a_{11}(1-\frac{X_1}{K})) + b_{31}X_1(1-\exp(-\beta_{31}X_3)) - c_{12}X_1(1-\exp(-\lambda_{21}X_2)) \quad (6)$$

The change in the above equation may be the 1st term. The other equations may be approximated by the same procedure. The Lyapunov exponent [12] in the present context is defined by,

$$\frac{dX_1(t+1)}{dX_1(t)}$$

for i = 1 to 4, corresponding to the 4 difference equations like (6) .

The Lyapunov exponent for (6) has been evaluated to be,

$$\frac{dX_1(t+1)}{dX_1(t)}$$
$$= a_{11}(1-\frac{X_1}{K}+\frac{1}{a_{11}})+b_{31}(1-\exp(-\beta_{31}X_3))$$
$$-c_{12}(1-\exp(-\lambda_{21}X_2)) \tag{7}$$

$dX_i(t+1)/dX_i(t)$ for other three difference equations are also determined similarly.

Now to the condition for chaos is given by

$$\frac{dX_1(t+1)}{dX_1(t)}>1. \tag{8}$$

Applying this condition to all the 4 difference equations determine the range of parameters of the dynamics tending to chaos.

In [9], we evaluated the range of parameters of the dynamics responsible for chaos.

3.1 Stability Analysis of the Proposed Emotional Dynamics by Lyapunov Energy Function:

Theorem 1: The dynamics (9) is asymptotically stable if,

$$a_{ii}X_i(1-\frac{X_i}{K})<\sum_{\exists j}b_{ji}X_i(1-\exp(-\beta_{ji}X_j)) \tag{10}$$

and L (x_i, x_j) is the Lyapunov function to prove its asymptotic stability.

$$\frac{dX_i}{dt}=a_{ii}X_i(1-\frac{X_i}{K})+\sum_{\exists j}b_{ji}X_i(1-\exp(-\beta_{ji}X_j)) \tag{9}$$

for i = 1 to n.

Let

$$L(X_i,X_j)=-\left((a_{ii}X_i^2-\frac{a_{ii}}{3K}X_i^3)-\sum_{j=1}^{n}\int_0^{X_i}b_{ji}X_i(1-\exp(\beta_{ji}X_j)\right) \tag{11}$$

Proof: We first show that L (x_i, x_j) is a Lyapunov energy function and then would like to verify that the time derivative of L for the given dynamics is unconditionally negative.

To prove that L (x_i, x_j) is a Lyapunov energy function, we verify that,

1. $L(0,0)=0$,
2. $\left(L(X_i,X_j)\right)_{X_i\neq 0,\, X_j\neq 0}>0$, when (10) is satisfied.
3. $\frac{\partial L}{\partial X_i}$, $\frac{\partial L}{\partial X_j}$ both exist.

This proves that L (x_i, x_j) is a Lyapunov energy function. Now, to show that dL/dt is unconditionally negative we evaluate:

$$\frac{dL}{dt}=\sum_{i=1}^{N}\frac{\partial L}{\partial X_i}\cdot\frac{dX_i}{dt}$$
$$=-[\sum_{i=1}^{n}a_{ii}X_i(1-\frac{X_i}{K})+\sum_{\exists j}b_{ji}X_i(1-\exp(-\beta_{ji}X_j))]^2<0 \tag{12}$$

Thus the theorem is proved.

When dx_i/dt contains co-operative terms like C_{ki} x_i (1-exp (- λ_{ik} x_k), for some n, the modified dynamics given by (1) can also be proved unconditionally stable by Theorem 2.

Theorem 2: Let L (x_i, x_j, x_k) given below be the Lyapunov function presented in Equation (1), where

L (x_i, x_j, x_k) =

$$-[(a_{ii}X^2-\frac{a_{ii}}{3K}X_i^3)-\sum_{j=1}^{n}\int_0^{X_i}b_{ji}X_i(1-\exp(\beta_{ji}X_j)$$
$$-\sum_{k=1}^{m}\int_0^{X_i}c_{ki}X_i(1-\exp(\lambda_{ik}X_k) \tag{13}$$

Dynamics (1) is conditionally stable using the above Lyapunov function when the following condition is satisfied.

$$a_{ii}X_i(1-\frac{X_i}{K})+\sum_{\exists j}b_{ji}X_i(1-\exp(-\beta_{ji}X_j))$$
$$<\sum_{\exists k}c_{ki}X_i(1-\exp(-\lambda_{ik}X_k)) \tag{14}$$

Proof: The proof is similar to the proof of Theorem 1.

4. Experiments and Results

The experiment includes submission of audio visual stimulus for arousal of emotions. In order to arouse mixed emotions, we need to submit time-staggered audio visual stimulus causing arousal of different emotions. In other words, persistence of more than one emotion can be maintained by arousing one base emotion, super imposed with other noisy emotions. For instance, a base emotion of happiness, when perturbed by a burst of disgust and fear in time-succession, represents a co-existence of multiple emotions for a certain time frame. After the mixed emotions are synthesized, due to submission of necessary stimulus in discrete time frame, three important facial attributes Mouth opening(MO), Eye opening(EO), and Eye brow's constriction (EBC) are determined from the facial expression of the subject. This needs segmentation of EO, MO, EBC region and their localization, the details [9],[14]-[16] of which are not discussed here for space limitation.

Table I
FEATURES OF EMOTIONS

Emotion	**Eye-open**	**Mouth–open**	**Eyebrow-const.**
DISGUST	SMALL	MEDIUM	**LARGE**
FEAR	**LARGE**	LARGE	SMALL
HAPPY	MEDIUM	**LARGE**	SMALL
ANXIOUS	MEDIUM	**SMALL**	**LARGE**
SAD	MEDIUM	**SMALL**	SMALL
ANGER	**LARGE**	**LARGE**	**LARGE**

General features of six emotions are given in table 1[9]. The primary feature for a given emotion is now determine experimentally with a large no ($\approx$1000) of subjects. It is noted that, for most of these subjects the primary emotions listed in the table [Bold entries] is supported by most ($\approx$862) of the subjects. Naturally, we determine, the primary features only to segregate different emotions from the manifestation in the facial expression of the subjects. For convenience of measurement, let us assume that the degree/strength of an emotion be proportional to its primary feature. Thus for happiness, denoted by state variable X_2 ($X_2 \propto$large mouth opening).Similarly, for Disgust, denoted by state variable X_1 ($X_1 \propto$for large eye brow's constriction). Lastly, for fear, denoted by state variable X_3 ($X_3 \propto$large eye opening). Since these primary features are all measurable, we can evaluate the parameters of the emotional dynamics: a_{ii}, b_{ji}, c_{ki} in (1) for stable and chaotic cases by respectively setting $\frac{dX_i}{dt} = 0$ and $\frac{dX_i(t+1)}{dX_i(t)} > 1$ for all i. the parameters λ_{ik}, β_{ji} and K are set empirically from computer simulations, and are not intervened while determining the dynamic behavior of emotions from experiments. Principles of parameter selection for the dynamics are illustrated here using Example 1 and 2.

Example 1: In figure 2, just after arousal of happiness, we have

$$\frac{dX_2}{dt} = a_{22} X_2 (1 - \frac{X_2}{K}) \quad (15)$$

Setting,

$\frac{dX_2}{dt} = X_2(t+1) - X_2(t)$, we have

$$X_2(t+1) = X_2(t)[1 + a_{22}(1 - \frac{X_2(t)}{K})]$$

Now, empirically [9], we have K=1000; And suppose, $X_2(1)$=13 pixels, $X_2(0)$= 10 pixels;

Now, we get a_{22}=0.303;

It is thus apparent from the example that for stable happiness we have a specific value of a_{22} for a given subject.

Example 2: When disgust appears, the above dynamics is modified to

$$\frac{dX_1}{dt} = a_{11} X_1 (1 - \frac{X_1}{K}) \quad (16)$$

$$\frac{dX_2}{dt} = a_{22} X_2 (1 - \frac{X_2}{K}) - c_{12} X_2 (1 - exp(-\lambda_{21} X_1))$$

(17) Reorganizing (16) and (17) we have,

$$X_2(t+1) = X_2(t)[1 + a_{22}(1 - \frac{X_2(t)}{K})] - c_{12} X_2 (1 - exp(-\lambda_{21} X_1))$$

(18) $X_1(t+1) = X_1(t)[1 + a_{11}(1 - \frac{X_1(t)}{K})]$ (19)

with λ_{21}=0.4 and K=1000 obtained empirically and $X_2(1) = 5, X_2(2) = 27$ obtained from measurements, we finally have from equations (18) and (19) at t=1,

$$\frac{X_2(2)}{X_2(1)} = 1 + a_{22}(1 - \frac{X_2(1)}{K}) - c_{12}(1 - \exp(-0.4 X_1(1))) \quad (20)$$

$$X_1(2) = X_1(1) + a_{11} X_1(1)(1 - \frac{X_1(1)}{K}) \quad (21)$$

Solving the above equations and setting a_{22} =0.303 as obtained from example1, we finally have c_{12} =3.3. It can be verified that the dynamics considered in example 1 and 2 do not support chaos as $\frac{dX_i(t+1)}{dX_i(t)}$ not greater than 1. However, when disgust, fear and happiness persist together as in sample 4 in figure 2, we obtain chaotic behavior of the dynamics. The parameters for the dynamics then are evaluated by setting , $\frac{dX_i(t+1)}{dX_i(t)} > 1$, for all the given equations at time t=6. The parameters a_{ii}, b_{ji}, c_{ki} obtained are given in table III. The λ_{ik}, β_{ji} and K are obtained empirically. Table II provides the parameters of facial extract for a subject excited with audio visual stimuli for 3 emotions as indicated in Figure 2. The parameters obtained for one stable dynamics, corresponding to example 2 and one chaotic situation (not presented for lack of space), corresponding to time slot 6(Figure 2) are given in table III and IV respectively. The corresponding phase portraits also indicate their stable and chaotic behavior. It is important to note that the dynamics (2)-(5) also exhibit Limit cyclic behavior for the parameters listed in table V and the phase a portrait corresponding to limit cycles is given in Figure 4. However, unfortunately in an experiment we did not notice Limit cycles for normal people. We presume that Limit cycles in dynamics may be seen in case of psychologically disabled people suffering from Trauma or epilepsy. This, however, needs more study and experiments with the dynamics.

Table II
PARAMETERS OF FACIAL EXTRACT OF SUBJECT (In pixels)

Features	T1	T2	T3	T4	T5	T6	T7	T8	T9
EO	17	13	12	15	19	6	18	7	16
MO	14	13	5	3	6	5	8	12	5
EBC	5	10	24	24	28	27	10	13	7

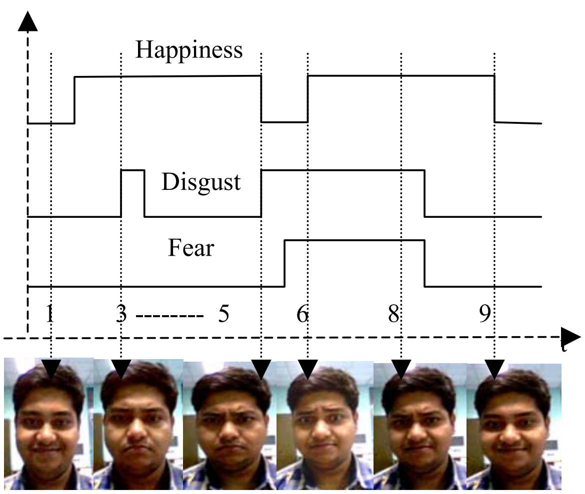

Figure 2. Arousal of different facial expressions of the subject is shown when different causal input is given to the subject.

Table III
PARAMETERS FOR CHAOS FROM CALCULATION

$\mathbf{a_{ii}}$	$\mathbf{b_{ji}}$	$\mathbf{c_{ki}}$	$\boldsymbol{\lambda_{ik}}$	$\boldsymbol{\beta_{ji}}$
a_{11}=0.38	b_{31}=0.9	c_{12}=0.65	λ_{21}=0.0045	β_{31}=0.006
a_{22}=0.0831	b_{12}=0.2	c_{23}=0.4	λ_{32}=0.007	β_{12}=0.001
a_{33}=0.245	b_{32}=0.3	c_{32}=0.5	λ_{23}=0.009	β_{32}=0.007
	b_{23}=0.4			β_{23}=0.005

Table IV
PARAMETERS FOR STABILITY FROM CALCULATION

$\mathbf{a_{ii}}$	$\mathbf{b_{ji}}$	$\mathbf{c_{ki}}$	$\boldsymbol{\lambda_{ik}}$	$\boldsymbol{\beta_{ji}}$
a_{11}=0.25	b_{31}=0.9	c_{12}=0.55	λ_{21}=0.0036	β_{31}=0.006
a_{22}=0.0831	b_{12}=0.2	c_{23}=0.4	λ_{32}=0.001	β_{12}=0.001
a_{33}=0.245	b_{32}=0.3	c_{32}=0.5	λ_{23}=0.005	β_{32}=0.001
	b_{23}=0.4			β_{23}=0.005

Table V
PARAMETERS FOR LIMIT CYCLE FROM CALCULATION

$\mathbf{a_{ii}}$	$\mathbf{b_{ji}}$	$\mathbf{c_{ki}}$	$\boldsymbol{\lambda_{ik}}$	$\boldsymbol{\beta_{ji}}$
a_{11}=0.03	b_{31}=0.65	c_{12}=0.55	λ_{21}=0.0035	β_{31}=0.005
a_{22}=0.0831	b_{12}=0.5	c_{23}=0.4	λ_{32}=0.005	β_{12}=0.001
a_{33}=0.245	b_{32}=0.3	c_{32}=0.5	λ_{23}=0.005	β_{32}=0.001
	b_{23}=0.5			β_{23}=0.005

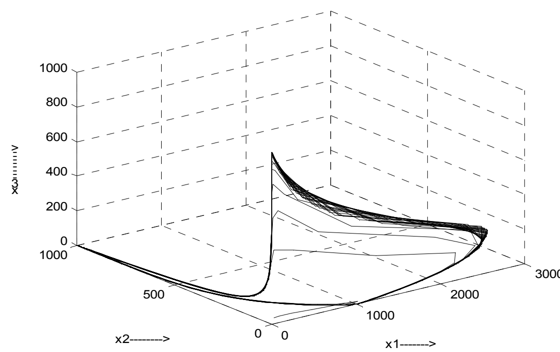

Figure 3. Chaotic behavior of Emotion dynamics

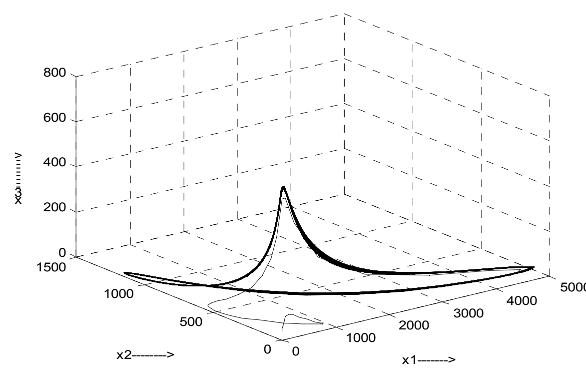

Figure 4: Limit cyclic behavior of Emotion dynamics

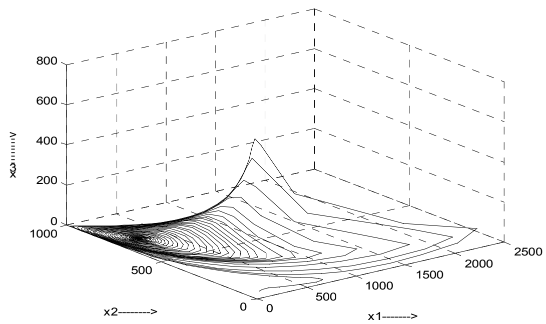

Figure 5: Stable behavior of Emotion dynamics

5. Conclusions

The paper proposed a new interactive model of emotional dynamics, and determined the conditions for selecting parametric range for stability and chaos of the dynamics. The principle of parameter selection of the dynamics for subjects excited with external stimulus are outlined and illustrated with numerical examples. The experimental study reveals that normal people exhibit both stable and chaotic behavior in their emotional exposition, but they never manifest limit cyclic behavior in their facial expression to indicate emotional fluctuation.

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